OSCILLATION OF FOURTH-ORDER DYNAMIC EQUATIONS

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Abstract
In this paper we shall reduce the problem of the oscillation of all solutions of certain nonlinear fourth-order dynamic equations to the problem of oscillation of two second-order dynamic equations, which are discussed intensively in the literature. Further oscillation criteria of fourth-order equations are given and proved using integration and Taylor’s formula on time scales. Some conditions are presented that ensure that all bounded solutions of the equation are oscillatory.

Keywords: Oscillation, Fourth-order, Dynamic equation, Time scales.


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1. Introduction
Consider the fourth-order nonlinear dynamic equation

\( x^{\Delta^4}(t) + q(t)x^{\lambda}(t) = 0, \)

where \( \lambda \) is the ratio of two positive odd integers and \( q \) is a real-valued positive and rd-continuous function on a time scale \( T \subset \mathbb{R} \) with \( \sup T = \infty \). Fourth-order differential equations (i.e., \( T = \mathbb{R} \)) and difference equations (i.e., \( T = \mathbb{N} \)) have been deeply investigated in the literature, see e.g., [7, 14, 16, 18, 21] for differential equations and [6, 13, 19, 22–24] for difference equations.

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