ON A CONNECTION BETWEEN THE
THEORY OF TACHIBANA OPERATORS
AND THE THEORY OF B-MANIFOLDS

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Abstract
The main purpose of the paper is to study the Tachibana operator for
a pure Riemannian metric tensor field and then to apply the results
obtained to the study of paraholomorphic B-manifolds.

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1. Introduction

Let $M_n$ be a Riemannian manifold with metric $g$, which is not necessarily positive
definite. We denote by $\mathcal{T}^p_q(M_n)$ the set of all tensor fields of type $(p, q)$ on $M_n$. Manifolds,
tensor fields and connections are always assumed to be differentiable and of class $C^\infty$.

An almost paracomplex manifold is an almost product manifold $(M_n, \varphi)$, $\varphi \in I$, such
that the two eigenbundles $T^+M_n$ and $T^-M_n$ associated with the two eigenvalues $+1$
and $-1$ of $\varphi$, respectively, have the same rank. Note that the dimension of an almost
paracomplex manifold is necessarily even.

Considering the paracomplex structure $\varphi$, we obtain the following set of affinors on $M_n$ :
$\{I, \varphi\}, \varphi^2 = I$, which form a basis of a representation of an algebra of order 2 over the
field of real numbers $\mathbb{R}$, which is called the algebra of paracomplex (or double) numbers
and is denoted by $\mathbb{R}(j) = \{a_0 + a_1j | j^2 = 1, a_0, a_1 \in \mathbb{R}\}$. Obviously, it is associative,
commutative and unital, i.e., it admits a principal unit 1. The canonical basis of this
algebra has the form $\{1, j\}$. The structural constants of an algebra are defined by the
multiplication law of the base units of this algebra: $e_ie_j = C^k_{ij}e_k$. With respect to the
canonical basis of $\mathbb{R}(j)$ the components of $C^i_{ij}$ are given by $C^1_{11} = C^2_{12} = C^2_{21} = C^2_{22} = 1$,
all the others being zero.

Consider $\mathbb{R}(j)$ endowed with the usual topology of $\mathbb{R}^2$ and a domain $U$ of $\mathbb{R}(j)$. Let
$X = x^1 + jx^2$