FOURIER METHOD FOR A QUASILINEAR PARABOLIC EQUATION WITH PERIODIC BOUNDARY CONDITION

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Abstract

A multidimensional mixed problem with Neuman type periodic boundary condition is studied for the quasilinear parabolic equation \( \frac{\partial u}{\partial t} - a^2 \frac{\partial^2 u}{\partial x^2} = f(t, x, u) \). The existence, uniqueness and also continuity of the weak generalized solution is proved.

Keywords: Quasilinear parabolic equation, Mixed problem, Fourier method, Periodic boundary condition, Generalized solutions.

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1. Introduction

In this study we consider the following mixed problem

\[
\begin{align*}
\frac{\partial u}{\partial t} - a^2 \frac{\partial^2 u}{\partial x^2} &= f(t, x, u), \quad (t, x) \in D := \{0 < t < T, \ 0 < x < \pi\} \\
u(t, 0) &= u(t, \pi), \quad t \in [0, T] \\
\frac{\partial u}{\partial x}(t, 0) &= \frac{\partial u}{\partial x}(t, \pi), \quad t \in [0, T] \\
u(0, x) &= \varphi(x), \quad x \in [0, \pi]
\end{align*}
\]

for a quasilinear parabolic equation with nonlinear source term \( f = f(t, x, u) \). Here \( a^2 = \frac{k}{\rho c} \), where \( k \) denotes the heat conduction coefficient, \( \rho \) denotes density and \( c \) specific heat.

The functions \( \varphi(x) \) and \( f(t, x, u) \) are given functions on \([0, \pi]\) and \( D \times (-\infty, \infty) \), respectively.

Denote by \( u = u(t, x) \) a solution of problem (1)-(4).

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