Hartley-Ross type unbiased estimators using the Stratified Random Sampling

Hatice Oncel Cekim∗† and Cem Kadilar‡

Abstract

This study mentions Hartley-Ross type unbiased ratio estimators of the finite population mean in the stratified random sampling using the auxiliary variable. We propose a class of unbiased estimators using the estimators in Kadilar and Cingi [5],[6]. We derive the variance equations, up to the first degree of approximation, for all proposed estimators. The proposed estimators have been compared with the mentioned estimators in theory. Finally, we also demonstrate theoretical findings by the support of numerical illustrations.

Keywords: ratio estimator; unbiased estimator; auxiliary information; efficiency; stratified random sampling; variance.

2000 AMS Classification: 62D05

1. Introduction and Notations

In the simple random sampling, Hartley and Ross [3] first defined the unbiased ratio estimator. Then, the unbiased ratio estimators in the stratified random sampling were presented by Pascual [10]. Singh et al. [11] and Kadilar and Cekim [4] proposed Hartley-Ross type unbiased estimators for the simple random sampling using various auxiliary information. Recently, Khan and Shabbir [7], [8] and Khan et al. [9] have also suggested several Hartley-Ross type unbiased estimators under the ranked set sampling and the stratified ranked set sampling.

A finite population $U = (U_1, U_2, ..., U_N)$ of size $N$ is assumed that the population of $N$ units be divided into $L$ strata with $N_h$ elements in the $h$–th stratum $(h = 1, 2, ..., L)$. Let $n_h$ be the size of the sample drawn by using the Simple Random Sampling without Replacement from a population of size $N_h$. Suppose that values $y_{hi}$ and $x_{hi}$ be on the study and auxiliary variables in the stratum $h$, respectively, where $i = 1, 2, ..., n_h$. Let the $h$–th stratum sample means be $\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{hi}$ and $\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{hi}$, respectively. Let the stratified mean estimator for $y$ and $x$ be , respectively, $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$. Here $W_h = (N_h/N)$ is the known stratum weight. The population means of

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the study and auxiliary variables are supposed that $Y = Y_{st} = \sum_{h=1}^{L} W_h Y_h$ and 
$X = X_{st} = \sum_{h=1}^{L} W_h X_h$, where $Y_h = \frac{1}{N_h} \sum_{i=1}^{N_h} Y_{hi}$ and $X_h = \frac{1}{N_h} \sum_{i=1}^{N_h} X_{hi}$, respectively.

The well-known ratio estimator of the population mean, $\bar{Y}$, is given by Cochran [1] as

\[(1.1) \quad y_C = \bar{Y} \bar{X},\]

where $\bar{Y} = \frac{\sum N_h}{N}$. Later, the bias of this estimator is estimated unbiasedly by Hartley and Ross [3] as

\[B(y_C) = -\frac{n(N-1)}{N(n-1)} (\bar{Y} - \bar{X})\]

and they obtain the unbiased ratio estimator

\[(1.2) \quad y_{HR} = \bar{Y} \bar{X} + \frac{n(N-1)}{N(n-1)} (\bar{Y} - \bar{X}),\]

for the population mean in the simple random sampling.

Kadilar and Cingi [5], [6] define some estimators using the coefficient of kurtosis ($\beta_2$) and the coefficient of variation ($C_x$) of the auxiliary variable under the stratified random sampling as

\[(1.3) \quad t_1 = \bar{Y}_{st} \frac{\bar{X}_{st} + C_{xst}}{\bar{x}_{st} + C_{xst}},\]

\[(1.4) \quad t_2 = \bar{Y}_{st} \frac{\bar{X}_{st} + \beta_{2st}(x)}{\bar{x}_{st} + \beta_{2st}(x)},\]

\[(1.5) \quad t_3 = \bar{Y}_{st} \frac{(\bar{X} \beta_2(x))_{st} + C_{xst}}{(\bar{x} \beta_2(x))_{st} + C_{xst}},\]

\[(1.6) \quad t_4 = \bar{Y}_{st} \frac{(\bar{X} C_x)_{st} + \beta_{2st}(x)}{(\bar{x} C_x)_{st} + \beta_{2st}(x)},\]

and

\[(1.7) \quad t_5 = k\frac{\bar{Y}_{st}}{\bar{x}_{st}} \bar{X},\]
where

\[ C_{xst} = \sum_{h=1}^{L} W_h C_{xh}, \quad \beta_{2st}(x) = \sum_{h=1}^{L} W_h \beta_{2h}(x), \]

\[ (\bar{X} \beta_2(x))_{st} = \sum_{h=1}^{L} W_h \bar{X}_h \beta_{2h}(x), \quad (\bar{x} \beta_2(x))_{st} = \sum_{h=1}^{L} W_h \bar{x}_h \beta_{2h}(x), \]

\[ (\bar{X} C_x)_{st} = \sum_{h=1}^{L} W_h \bar{X}_h C_{xh}, \quad (\bar{x} C_x)_{st} = \sum_{h=1}^{L} W_h \bar{x}_h C_{xh}, \]

and \( k \) is a constant that makes the mean squared error (MSE) of \( t_5 \) minimum.

The biases of the estimators, in (1.3)-(1.7), are obtained, to the first degree of approximation, respectively, as follows:

\[
B(t_j) = \frac{1}{X_{Sj}} \sum_{h=1}^{L} W_h^2 \gamma_h \left[ \frac{Y_{st}^2}{X_{Sj}} - S_{xhx}^2 - S_{yxh} \right], \quad j = 1, 2, 3, 4
\]

and

\[
B(t_5) = (k - 1) \bar{Y} + \frac{1}{X} \left[ \sum_{h=1}^{L} W_h^2 \gamma_h \left( \frac{Y_{st}^2}{X} - S_{xhx}^2 - kS_{yxh} \right) \right],
\]

such that

\[
S_{yhx}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2, \quad S_{xhx}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2,
\]

\[
S_{yxh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)(x_{hi} - \bar{X}_h) \quad \text{and} \quad \gamma_h = \frac{N_h - n_h}{N_h n_h}
\]

where

\[
X_{S1} = \bar{X}_{st} + C_{xst}, \quad X_{S2} = \bar{X}_{st} + \beta_{2st}(x),
\]

\[
X_{S3} = (\bar{X} \beta_2(x))_{st} + C_{xst} \quad \text{and} \quad X_{S4} = (\bar{X} C_x)_{st} + \beta_{2st}(x).
\]

2. Proposed Estimators

We improve Hartley-Ross estimators using the proposed estimators by Kadilar and Cingi [5], [6] with their unbiased biases, and in this way, we obtain the following
estimators:

\[ y_{\text{New}1} = \bar{y}_{st} \frac{X_{S1}}{\bar{x}_{st} + C_{xst}} \]

\[ y_{\text{New}2} = \frac{X_{S2}}{\bar{y}_{st} + \beta_{2st}(x)} \]

\[ y_{\text{New}3} = \frac{X_{S3}}{\bar{y}_{st} (\bar{\beta_2(x)})_{st} + C_{xst}} \]

\[ y_{\text{New}4} = \frac{X_{S4}}{\bar{y}_{st} (\bar{x_2(x)})_{st} + \beta_{2st}(x)} \]

\[ y_{\text{New}5} = k \bar{y}_{st} \bar{x} - (k - 1) \bar{y}_{st} - \frac{1}{\bar{X}} \left[ \sum_{h=1}^L W_h^2 \gamma_h \left( \frac{\bar{y}_{st}}{X_{S1}} S_{xh}^2 - s_{yxh} \right) \right] \]

where \( \bar{y}_{st} \) and \( s_{yxh} \) are unbiased estimators of \( \bar{Y}_{st} \) and \( S_{yx} \), respectively.

To obtain the variance of the suggested estimators, we define

\[ \bar{y}_{st} = \bar{Y} (1 + \vartheta_0), \bar{x}_{st} = \bar{X} (1 + \vartheta_1), \text{ and } \sum_{h=1}^L W_h^2 \gamma_h s_{yxh} = \sum_{h=1}^L W_h^2 \gamma_h S_{yx}(1 + \vartheta_2) \]

such that

\[ E(\vartheta_0) = E(\vartheta_1) = E(\vartheta_2) = 0, \]

\[ E(\vartheta_0^2) = V_{0,2}, E(\vartheta_1^2) = V_{2,0}, E(\vartheta_2^2) = D_{0,0}, \]

\[ E(\vartheta_0 \vartheta_1) = V_{1,1}, E(\vartheta_0 \vartheta_2) = D_{0,1} \text{ and } E(\vartheta_1 \vartheta_2) = D_{1,0}, \]

where

\[ V_{r,s} = \sum_{h=1}^L W_h^{r+s} \frac{E \left[ (\bar{x}_h - \bar{X}_h)^r (\bar{y}_h - \bar{Y}_h)^s \right]}{\bar{X}^r \bar{Y}^s}, \]
\[ D_{r,s} = \sum_{h=1}^{L} W_h^{r+s+1} \gamma_h \left( \frac{S_{yxh}^2}{W_h \gamma_h (\mu_{2kh} - S_{yxh}^2)} \right)^{r+s-1} \sum_{h=1}^{L} W_h^{r+s} \gamma_h \mu_{21h}^r \mu_{21h}^s, \]

and

\[ \mu_{jkh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (Y_{hi} - \bar{Y}_h)^2 (X_{hi} - \bar{X}_h)^k, \quad h = 1, 2, \ldots, L. \]

We express the proposed estimators \( y_{Newi}, i = 1, 2, \ldots, 5 \) with regard to \( \vartheta \)'s as:

\[ y_{New1} = \bar{Y}(1 + \vartheta_0)(1 + \alpha \vartheta_1)^{-1} \]
\[-\frac{1}{X_{S1}} \left[ \sum_{h=1}^{L} W_h^{2} \gamma_h \left( \frac{\bar{Y}(1 + \vartheta_0)}{X_{S1}} S_{xh}^2 - S_{yxh} (1 + \vartheta_2) \right) \right], \]

\[ y_{New2} = \bar{Y}(1 + \vartheta_0)(1 + \delta \vartheta_1)^{-1} \]
\[-\frac{1}{X_{S2}} \left[ \sum_{h=1}^{L} W_h^{2} \gamma_h \left( \frac{\bar{Y}(1 + \vartheta_0)}{X_{S2}} S_{xh}^2 - S_{yxh} (1 + \vartheta_2) \right) \right], \]

\[ y_{New3} = \bar{Y}(1 + \vartheta_0)(1 + \varphi \vartheta_1)^{-1} \]
\[-\frac{1}{X_{S3}} \left[ \sum_{h=1}^{L} W_h^{2} \gamma_h \left( \frac{\bar{Y}(1 + \vartheta_0)}{X_{S3}} S_{xh}^2 - S_{yxh} (1 + \vartheta_2) \right) \right], \]

\[ y_{New4} = \bar{Y}(1 + \vartheta_0)(1 + w \vartheta_1)^{-1} \]
\[-\frac{1}{X_{S4}} \left[ \sum_{h=1}^{L} W_h^{2} \gamma_h \left( \frac{\bar{Y}(1 + \vartheta_0)}{X_{S4}} S_{xh}^2 - S_{yxh} (1 + \vartheta_2) \right) \right], \]

and

\[ y_{New5} = k\bar{Y}(1 + \vartheta_0)(1 + \vartheta_1)^{-1} - (k - 1) \bar{Y}(1 + \vartheta_0) \]
\[-\frac{1}{X} \left[ \sum_{h=1}^{L} W_h^{2} \gamma_h \left( RS_{xh}^2 (1 + \vartheta_0) - k S_{yxh} (1 + \vartheta_2) \right) \right], \]

where

\[ \alpha = \frac{X_{S1}}{X_{S1}}, \quad \delta = \frac{X_{S2}}{X_{S2}}, \quad \varphi = \frac{(X_{S3} \beta_2(x))_{st}}{X_{S3}}, \quad \text{and} \quad w = \frac{(X_{S4} C)_{st}}{X_{S4}}. \]

In this way, we obtain the variance equations of the proposed estimators that are given in (2.1)-(2.5), respectively, as follows:
\[
V(y_{\text{New}}) \cong Y^2 A_\theta + \frac{1}{X S_j} \left[ \sum_{h=1}^L W_h^2 \gamma_h \left( \frac{Y S_{xh}^2 - S_{yxh}}{X S_j} \right) \right]^2 
\]
(2.6)
\[
- \frac{2Y}{X S_j} \left[ \sum_{h=1}^L W_h^2 \gamma_h \left( \frac{Y S_{xh}^2 A_\theta - S_{yxh} B_\theta \theta}{X S_j} \right) \right], \quad j = 1, 2, 3, 4
\]
and
\[
V(y_{\text{New5}}) \cong Y^2 A_\theta + \frac{1}{X^2} \left[ \sum_{h=1}^L W_h^2 \gamma_h \left( R S_{xh}^2 - k S_{yxh} \right) \right]^2 
\]
(2.7)
\[
-2R \left[ \sum_{h=1}^L W_h^2 \gamma_h \left( R S_{xh}^2 (A_\theta + k (1 - k) V_{2,0}) \right) \right],
\]
where
\[
A_\theta = V_{0,2} + \theta^2 V_{2,0} - 2\theta V_{1,1}, \quad R = \frac{Y}{X},
\]
and
\[
B_\theta = -\theta V_{1,1} + D_{0,1} - \theta V_{1,0} + \theta^2 V_{2,0}, \quad \theta = \alpha, \delta, \varphi, w \text{ and } k.
\]

Note that the term of \(\gamma^3_h\) is ignored, because it is equal to approximately zero. For minimizing the variance, given in (2.7), we obtain the optimum value of \(k\) by
(2.8)
\[
k_{\text{opt}} = \frac{\Delta}{\Pi}
\]
where
\[
\Delta = Y^2 V_{1,1} + \frac{1}{X^2} \left[ \sum_{h=1}^L W_h^2 \gamma_h^2 R S_{xh}^2 S_{yxh} + \sum_{h=1}^L \sum_{t=1}^L W_h^2 \gamma_h W_t^2 \gamma_t R (S_{xh}^2 S_{yxt} + S_{xt}^2 S_{yhx}) \right] 
\]
\[+ R \left[ \sum_{h=1}^L W_h^2 \gamma_h \left( R S_{xh}^2 (-2V_{1,1} + V_{2,0}) - S_{yxh} D_{0,1} \right) \right],
\]
and
\[
\Pi = Y^2 V_{2,0} + \frac{1}{X^2} \left[ \sum_{h=1}^L W_h^2 \gamma_h S_{yxh} \right]^2 
\]
\[+ 2R \sum_{h=1}^L W_h^2 \gamma_h S_{yxh} (-D_{1,0} - V_{1,1} + V_{2,0}).
\]
Replacing this optimum value in (2.7), to make the \(V(y_{\text{New5}})\) minimum, we get
(2.9)
\[
V_{\text{min}}(y_{\text{New5}}) \cong \Gamma - \frac{\Delta^2}{\Pi},
\]
where
\[
\Gamma = V_{0,2} \left[ Y^2 - 2R^2 \sum_{h=1}^{L} W_h^2 \gamma_h S_x^2 \right] + \frac{R^2}{X^2} \left[ \sum_{h=1}^{L} W_h^2 \gamma_h S_x^2 \right].
\]

3. Efficiency Comparisons

In this section, we compare proposed unbiased estimators given in (2.1)-(2.5), with the mentioned estimators, given in (1.3)-(1.7). Firstly, comparing the variance of the proposed estimators in (2.6) with the MSE of the estimators given in Kadilar and Cingi [5], we have the following inequality
\[
V(y_{Newj}) < MSE(t_j) = Y^2 A_\theta, \text{ where } \theta = \alpha, \delta, \varphi, \text{ and } j = 1, 2, 3, 4,
\]
if
\[
\frac{2Y}{X_{Sj}} \left[ \sum_{h=1}^{L} W_h^2 \gamma_h \left( \frac{Y S_{xh}}{X_{Sj}} A_\theta - S_{yxh} B_\theta \right) \right] - \frac{1}{X_{Sj}^2} \left[ \sum_{h=1}^{L} W_h^2 \gamma_h \left( \frac{Y S_{xh}}{X_{Sj}} - S_{yxh} \right) \right]^2 < 0, \quad j = 1, 2, 3, 4.
\]

Secondly, comparing the variance of the proposed estimator in (2.7) with the MSE of the estimators given in Kadilar and Cingi [6], we have
\[
V(y_{New5}) < MSE(t_5) = Y^2 \left\{ k^* C + (k^* - 1)^2 \right\},
\]
where
\[
C = V_{2,0} - 2V_{1,1} + V_{0,2} \text{ and } \theta = k,
\]
if
\[
Y^2 \left[ k^* C + (k^* - 1)^2 - A_\theta \right] - 2R \left[ \sum_{h=1}^{L} W_h^2 \gamma_h \left( R S_{xh}^2 \left( A_\theta + k (1 - k) V_{2,0} \right) \right) \right. \\
- kS_{yxh} \left( B_\theta + k (1 - k) V_{2,0} \right)] \\
+ \frac{1}{X^2} \left[ \sum_{h=1}^{L} W_h^2 \gamma_h \left( R S_{xh}^2 - k S_{yxh} \right) \right]^2 < 0.
\]

Finally, we also compare the minimum variance of the proposed estimator in (2.9) with the minimum MSE of the estimators given in Kadilar and Cingi [6]. For this reason, it can be written as
\[
V_{min} (y_{New5}) < MSE_{min}(t_5) = Y^2 \frac{C}{C + 1}
\]
if

\[
Y^2 \frac{C}{C+1} - \left[ \Gamma - \frac{\Delta^2}{\Pi} \right] < 0,
\]

where the optimum value of \( k^* \) is

\[
k^*_{\text{opt}} = \frac{1}{C+1}.
\]

If the conditions (3.1)-(3.3) are satisfied, the proposed estimators are more efficient than the mentioned estimators \( t_i, i = 1, 2, \ldots, 5 \), under the determined conditions.

4. Empirical Study

To show the merits of the proposed estimators among the other estimators, two data sets previously used by Kadilar and Cingi [5] and Cingi et al. [2] are considered. First data set consists of 854 districts in Turkey. Summaries of the Population I are shown in Table 1.

Population I (Source: Institute of Statistics, Republic of Turkey [5]):

\( Y \); the apple production amount in 1999, \( X \); the number of apple trees in 1999. Stratum: Regions in Turkey (as 1: Marmara; 2: Aegean; 3: Mediterranean; 4: Central Anatolia; 5: Black Sea; 6: East and Southeast Anatolia).

Second data set consists of 923 districts in Turkey. Similarly, summaries of the Population II are shown in Table 2.

Population II (Source: Ministry of Education, Republic of Turkey [2]):

\( Y \); the number of students in 2007, \( X \); the number of schools in 2007. Stratum: Regions in Turkey (as 1: Marmara; 2: Aegean; 3: Mediterranean; 4: Central Anatolia; 5: Black Sea; 6: East and Southeast Anatolia).

<table>
<thead>
<tr>
<th>Stratum</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_h )</td>
<td>106</td>
<td>106</td>
<td>94</td>
<td>171</td>
<td>204</td>
<td>173</td>
</tr>
<tr>
<td>( n_h )</td>
<td>9</td>
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<td>38</td>
<td>67</td>
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<tr>
<td>( W_h )</td>
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<td>0.12</td>
<td>0.11</td>
<td>0.20</td>
<td>0.24</td>
<td>0.20</td>
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<tr>
<td>( \bar{Y}_h )</td>
<td>1336</td>
<td>2212</td>
<td>9384</td>
<td>5588</td>
<td>967</td>
<td>404</td>
</tr>
<tr>
<td>( \bar{X}_h )</td>
<td>24375</td>
<td>27421</td>
<td>72409</td>
<td>74365</td>
<td>26441</td>
<td>9844</td>
</tr>
<tr>
<td>( C_{xh} )</td>
<td>2.02</td>
<td>2.10</td>
<td>2.22</td>
<td>3.84</td>
<td>1.72</td>
<td>1.91</td>
</tr>
<tr>
<td>( \beta_{2h}(x) )</td>
<td>26.68</td>
<td>34.57</td>
<td>26.14</td>
<td>97.6</td>
<td>27.47</td>
<td>28.11</td>
</tr>
<tr>
<td>( S_{yh} )</td>
<td>6425</td>
<td>11552</td>
<td>29907</td>
<td>28643</td>
<td>2390</td>
<td>946</td>
</tr>
<tr>
<td>( S_{xh} )</td>
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<td>57461</td>
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<td>285603</td>
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<td>18794</td>
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</table>

Second data set consists of 923 districts in Turkey. Similarly, summaries of the Population II are shown in Table 2.
Table 2. Descriptive Statistics for the Population II

<table>
<thead>
<tr>
<th>Stratum</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_h$</td>
<td>127</td>
<td>117</td>
<td>103</td>
<td>170</td>
<td>205</td>
<td>201</td>
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<tr>
<td>$n_h$</td>
<td>31</td>
<td>21</td>
<td>29</td>
<td>38</td>
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<td>39</td>
</tr>
<tr>
<td>$W_h$</td>
<td>0.14</td>
<td>0.13</td>
<td>0.11</td>
<td>0.18</td>
<td>0.22</td>
<td>0.22</td>
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<tr>
<td>$V_h$</td>
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<td>9211.79</td>
<td>14309.30</td>
<td>9478.85</td>
<td>5569.95</td>
<td>12997.59</td>
</tr>
<tr>
<td>$X_h$</td>
<td>30.81</td>
<td>30.29</td>
<td>43.19</td>
<td>30.21</td>
<td>29.50</td>
<td>57.54</td>
</tr>
<tr>
<td>$C_{xh}$</td>
<td>0.85</td>
<td>0.83</td>
<td>1.09</td>
<td>1.01</td>
<td>0.99</td>
<td>0.84</td>
</tr>
<tr>
<td>$\beta_{2h}(x)$</td>
<td>2.51</td>
<td>2.09</td>
<td>8.42</td>
<td>3.49</td>
<td>4.07</td>
<td>8.2</td>
</tr>
<tr>
<td>$S_{yh}$</td>
<td>30486.75</td>
<td>15180.77</td>
<td>27549.70</td>
<td>18218.93</td>
<td>8497.78</td>
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<td>$S_{xh}$</td>
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<td>25.08</td>
<td>47.12</td>
<td>30.4</td>
<td>29.33</td>
<td>48.26</td>
</tr>
</tbody>
</table>

The sample sizes of each stratum are selected with the help of the Neyman allocation method for two data sets. From Table 3, we infer that proposed estimators have the smaller MSE values than the corresponding estimators in literature, and therefore, the proposed estimators are more efficient than the estimators existed in literature for two population data sets I and II.

Table 3. MSE and Variance values of $t_j$ and $y_{\text{New}j}$ ratio estimators

<table>
<thead>
<tr>
<th>Population I</th>
<th>Estimators</th>
<th>$MSE$</th>
<th>Estimators</th>
<th>$Var$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
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<td>$y_{\text{New}1}$</td>
<td>191806.32</td>
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<td>$t_2$</td>
<td>214105.63</td>
<td>$y_{\text{New}2}$</td>
<td>191936.66</td>
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<td>$t_3$</td>
<td>213976.29</td>
<td>$y_{\text{New}3}$</td>
<td>191798.91</td>
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5. Conclusion

In this article, we study on the estimators given by Kadilar and Cingi [5], [6] to obtain the unbiased estimation of the population mean in the stratified random sampling. Both the theoretical and empirical results show that the suggested unbiased estimators have smaller variance values than the compared estimators under the determined conditions. Moreover, the results in Table 3 clearly indicate that the suggested estimator of $y_{\text{New}5}$ is the best estimator for the data sets used in Section 4.
References


