CONSEQUENCES OF ALLEE EFFECTS ON STABILITY ANALYSIS OF THE POPULATION MODEL $X_{t+1} = \lambda X_t f(X_{t-3})$

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Abstract

The stability conditions of equilibrium points of the population model $X_{t+1} = \lambda X_t f(X_{t-3})$ with and without Allee effects are investigated. It is assumed that the Allee effect occurs at low population density. Analysis and numerical simulations show that Allee effects have both stabilizing and destabilizing effects on population dynamics with delay.

Keywords: Stability, Allee effect, Population model, Equilibrium point, Difference equation, Time delay.

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1. Introduction.

Studies on population dynamics have been attractive for ecologists, biologists and mathematicians. Today, many of the population phenomenon are modelled by scientists using discrete and continuous dynamical systems. Although a discrete-time population model represents a richer dynamical picture (specifically, in terms of numerical simulation), continuous-time population models are more appropriate to nature except for non-overlapping generations. These models sometimes consist of a delay term if they depend on past history.

Formerly, scientists believed that a population achieves its equilibrium density when everything is sufficient (food, place, finding mates, etc.). However, Allee in 1931 [1] introduced a new idea which represents a negative density dependence when the population growth rate is reduced to a low population size. This may be due to a number of sources including difficulties in finding mates, social dysfunction at small population sizes, inbreeding depression, food exploitation, predator avoidance of defence. Such effects may be observed on different organisms including vertebrates, invertebrates and plants. The

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