RECONSTRUCTION OF COMPLEX JACOBI MATRICES FROM SPECTRAL DATA

Gusein Sh. Guseinov

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Abstract

In this paper, we introduce spectral data for finite order complex Jacobi matrices and investigate the inverse problem of determining the matrix from its spectral data. Necessary and sufficient conditions for the solvability of the inverse problem are established. An explicit procedure of reconstruction of the matrix from the spectral data is given.

Keywords: Jacobi matrix, Difference equation, Spectral data, Inverse spectral problem.

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1. Introduction

Inverse spectral problems for Jacobi matrices play an important role in the study of nonlinear discrete dynamical systems such as Toda lattices [4, 14, 16, 17]. Also, Jacobi matrices are known to be a very useful tool in the study of the moment problem, orthogonal polynomials, Padé approximation, and Jacobi continued fractions [1, 3, 7, 15, 18].

Consider a general $N \times N$ complex, symmetric, tri-diagonal matrix – a Jacobi matrix:

\[
J = \begin{bmatrix}
    b_0 & a_0 & 0 & \cdots & 0 & 0 & 0 \\
    a_0 & b_1 & a_1 & \cdots & 0 & 0 & 0 \\
    0 & a_1 & b_2 & \cdots & 0 & 0 & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
    0 & 0 & 0 & \cdots & b_{N-3} & a_{N-3} & 0 \\
    0 & 0 & 0 & \cdots & a_{N-3} & b_{N-2} & a_{N-2} \\
    0 & 0 & 0 & \cdots & 0 & a_{N-2} & b_{N-1}
\end{bmatrix},
\]

where for each $n$, $a_n$ and $b_n$ are arbitrary complex numbers such that $a_n$ is different from zero:

\[
a_n, \ b_n \in \mathbb{C}, \ a_n \neq 0.
\]

*Department of Mathematics, Atılım University, 06836 Incek, Ankara, Turkey.
E-mail: guseinov@atilim.edu.tr