ON GENERALIZED FIBONACCI
AND LUCAS NUMBERS
BY MATRIX METHODS

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Abstract
In this study we define the generalized Lucas $V(p, q)$-matrix similar to the generalized Fibonacci $U(1, -1)$-matrix. The $V(p, q)$-matrix is different from the Fibonacci $U(p, q)$-matrix, but is related to it. Using this matrix representation, we have found some well-known equalities and a Binet-like formula for the generalized Fibonacci and Lucas numbers.

Keywords: Matrix methods, generalized Lucas numbers, Binet’s formula.

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1. Introduction
Consider a sequence $\{W_n\} = \{W_n(a, b, p, q)\}$ defined by the recurrence relation

$$W_n = pW_{n-1} - qW_{n-2}, \quad n \geq 2,$$

with $W_0 = a, W_1 = b$, where $a, b, p$ and $q$ are integers with $p > 0, q \neq 0$.

We are interested in the following two special cases of $\{W_n\}$: $\{U_n\}$ is defined by $U_0 = 0, U_1 = 1$, and $\{V_n\}$ is defined by $V_0 = 2, V_1 = p$. It is well known that $\{U_n\}$ and $\{V_n\}$ can be expressed in the form

$$U_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, \quad V_n = \alpha^n + \beta^n,$$

where $\alpha = \frac{p + \sqrt{\Delta}}{2}, \beta = \frac{p - \sqrt{\Delta}}{2}$ and the discriminant is $\Delta = p^2 - 4q$.

Especially, if $p = -q = 1$ and $2p = -q = 2$, $\{U_n\}$ is the usual Fibonacci and Jacobsthal sequence, respectively.

We define $U(p, q)$ be the $2 \times 2$ matrix

$$U(p, q) = \begin{bmatrix} p & -q \\ 1 & 0 \end{bmatrix},$$

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