GENERAL APPROACH IN COMPUTING
SUMS OF PRODUCTS OF BINARY
SEQUENCES

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Received 10:02:2010 : Accepted 16:12:2011

Abstract
In this paper we find a general approach to find closed forms of sums
of products of arbitrary sequences satisfying the same recurrence with
different initial conditions. We apply successfully our technique to sums
of products of such sequences with indices in (arbitrary) arithmetic
progressions. It generalizes many results from literature. We propose
also an extension where the sequences satisfy different recurrences.

Keywords: Second order recurrences; Sums; Products
2000 AMS Classification: 11B37, 11C20

1. Introduction
We consider a generic (nondegenerate, that is, \( \delta = \sqrt{p^2 - 4q} \neq 0 \)) binary recurrence
satisfying
\[
X_{n+1} = pX_n - qX_{n-1}, \quad n \in \mathbb{Z}
\]
with some initial conditions. Let \( \alpha, \beta \) be the roots of the equation \( x^2 - px + q = 0 \), and
so, \( \alpha + \beta = p, \alpha\beta = q, \delta = \alpha - \beta \). We associate the companion Lucas sequence \( L_n \)
which also satisfies (1.1) together with \( L_0 = 2, L_1 = p \), and so \( L_n = \alpha^n + \beta^n \).

Let \( \{U_{n}^{(j)}\}_{j=1}^{p} \) be a set of \( p \) binary sequences, all of which will satisfy the recurrence
(1.1) with some initial conditions, such that the Binet formula for these sequences is
\[
U_{n}^{(j)} = A_j \alpha^n + B_j \beta^n, \quad n \in \mathbb{Z},
\]
where \( A_j = \frac{u^{(j)}_0 - u^{(j)}_1 \delta}{\delta}, \quad B_j = \frac{u^{(j)}_0 - u^{(j)}_1 \alpha}{\delta} \).

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