

CONSTRUCTION OF A COMPLEX JACOBI MATRIX FROM TWO-SPECTRA

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Abstract

In this paper we study the inverse spectral problem for two-spectra of finite order complex Jacobi matrices (tri-diagonal matrices). The problem is to reconstruct the matrix using two sets of eigenvalues, one for the original Jacobi matrix and one for the matrix obtained by deleting the first column and the first row of the Jacobi matrix. An explicit procedure of reconstruction of the matrix from the two-spectra is given.

Keywords: Jacobi matrix, Spectral data, Inverse spectral problem.

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1. Introduction

An $N \times N$ complex Jacobi matrix is a matrix of the form

$$(1.1) \quad J = \begin{bmatrix} b_0 & a_0 & 0 & \cdots & 0 & 0 & 0 \\ a_0 & b_1 & a_1 & \cdots & 0 & 0 & 0 \\ 0 & a_1 & b_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & b_{N-3} & a_{N-3} & 0 \\ 0 & 0 & 0 & \cdots & a_{N-3} & b_{N-2} & a_{N-2} \\ 0 & 0 & 0 & \cdots & 0 & a_{N-2} & b_{N-1} \end{bmatrix},$$

where for each n , a_n and b_n are arbitrary complex numbers such that a_n is different from zero:

$$(1.2) \quad a_n, b_n \in \mathbb{C}, a_n \neq 0.$$

The general inverse spectral problem is to reconstruct the matrix given some of its spectral characteristics (spectral data).

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