CHARACTERIZATIONS OF REAL DIFUNCTIONS

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Abstract

The notion of difunction between textures has proved to be of considerable interest and importance. In this paper the authors consider real difunctions, that is difunctions from a given texture \((S, S)\) to the real texture \((\mathbb{R}, \mathbb{R})\), and seek representations of such difunctions in terms of ordinary point functions. It is shown that in general real difunctions cannot be represented in terms of real-valued point functions on \(S\), but that they can be represented by real-valued point functions on the core \(S^0\) of \(S\). Equivalently, it is shown that instead of restricting to the core of \(S\), the real texture \((\mathbb{R}, \mathbb{R})\) may be replaced by the extended real texture \((\mathbb{R}^+, \mathbb{R}^+)\) and representations obtained in terms of point functions from \(S\) to \(\mathbb{R}^+\).

Keywords: Texture, Ditopology, Real texture, Real difunction, Point function, Interval-valued function, Bicontinuity, Extended real texture.

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1. Introduction

There is now a considerable literature on the theory of ditopological texture spaces, and an adequate introduction to this theory and the motivation for its study may be obtained from \([2, 3, 4, 5, 6, 7]\).

For a texture \((S, S)\), most properties are conveniently defined in terms of the \(p\)-sets \(P_s = \bigcap\{A \in S \mid s \in A\}\) and the \(q\)-sets \(Q_s = \bigvee\{A \in S \mid s \not\in A\}\). However, as noted in \([1]\) we may associate with \((S, S)\) the C-space (core-space) \([9, 10, 11, 13, 14]\) \((S, S^0)\), and then the frequently occurring relationship \(P_s \not\subseteq Q_s, s, s' \in S\), is equivalent to \(s \omega S s'\), where \(\omega S\) is the interior relation for \((S, S^0)\). In this paper we will use whichever notation seems to be the more convenient in each particular instance.

We will be especially interested in the real texture \((\mathbb{R}, \mathbb{R})\), where \(\mathbb{R}\) denotes the set of real numbers and \(\mathcal{R}\) is the texturing \(\{(-\infty, r) \mid r \in \mathbb{R}\} \cup \{(\infty, r) \mid r \in \mathbb{R}\} \cup \{\mathbb{R}, \emptyset\}\).

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