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APPROXIMATION BY q-PHILLIPS OPERATORS

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Abstract

In this study, we introduce a q-analogue of the Phillips operators and investigate approximation properties. We establish direct and local approximation theorems. We give a weighted approximation theorem. We estimate the rate of convergence of these operators for functions of polynomial growth on the interval $[0, \infty)$.

Keywords: Phillips operators, q-type operators, Rate of convergence, Weighted approximation, q-integral.

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1. Introduction

Phillips firstly introduced the q-analogue of Bernstein polynomials based on q-integer and q-binomial coefficients in [12]. Gupta and Finta obtain some direct results on certain q-Durrmeyer type operators in [6]. Recently, Aral and Gupta introduced Durrmeyer type modification of the q-Baskakov type operators in [1]. We aim to introduce a q-analogue of Phillips operators and to study approximation properties. Before this, we mention the following notations and formulas, which can be founded in [2, 8, 9] and [10]: For $n \in \mathbb{N}$, $0 < q < 1$ and $a, b \in \mathbb{R}$,

$$[n]_q = 1 + q + q^2 + \cdots + q^{n-1},\ n \in \mathbb{N}\backslash\{0\};\ [0]_q = 0, \tag{1.1}$$

$$[n]_q! = [1]_q[2]_q \cdots [n]_q,\ n \in \mathbb{N}\backslash\{0\};\ [0]_q! = 1, \tag{1.2}$$

$$(a+b)_q^n = \prod_{j=1}^{n}(a + q^{j-1}b), \tag{1.3}$$

and

$$(1+a)_q^\infty = \prod_{j=1}^{\infty}(1 + q^{j-1}a). \tag{1.4}$$

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