MEANS OF FP-SOFT SETS AND THEIR APPLICATIONS

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Abstract
Soft set theory was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with problems that contain uncertainties. In this paper, we first give most of the fundamental definitions of FP-soft set theory. We secondly define means of FP-soft sets and study their related properties. We then define decision making methods on FP-soft set theory. We finally apply the method successfully to problems that contain fuzzy objects.

Keywords: Soft sets, Fuzzy sets, FP-soft sets, AND-means, OR-means, Decision making.

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1. Introduction
In 1999, the concept of soft sets was introduced by Molodtsov [22] for modeling problems that contain vagueness and uncertainty. After Molodtsov, Maji et al. [20] gave the operations of soft sets and their properties. Since then, based on these operations, soft set theory has developed in many directions and found its applications in a wide variety of fields. For instance; on the theory of soft sets [2, 4, 5, 9, 17, 20, 21, 25], on soft decision making [13, 14, 15, 18, 19, 24], on fuzzy soft sets [7, 10, 11], soft rough sets [13] are some of the selected works. Some authors, such as [1, 3, 6, 16, 23, 26, 27] have also studied the algebraic properties of soft sets.

The present expository paper is a condensation of part of the dissertation [12]. The presentation of the rest of the paper is organized as follows. In the next section, most of the fundamental definitions of the operations of fuzzy sets and soft sets are presented. In Section 3, we give FP-soft set-sets and their operations, which are more functional, to make theoretical studies of soft set theory in greater detail and improve several results.

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In Section 4, we define means of FP-soft sets and their properties. In Section 5, we define soft fuzzification operators for And means and Or means of two FP-soft sets. In Section 6, we construct a FP-soft set-decision making method. We also give an application which shows that these methods work successfully. In the final section, some concluding comments are presented.

2. Preliminary

In this section, we present the basic definitions and results of soft set theory [22] and fuzzy set theory [28] that are useful for subsequent discussions. Detailed explanations related to soft sets and fuzzy sets can be found in [4, 20, 22] and [28, 29], respectively.

2.1. Definition. [22] Let $U$ be an initial universe, $P(U)$ the power set of $U$, $E$ a set of parameters and $A \subseteq E$. Then, a soft set $F_A$ over $U$ is defined as follows:

$$F_A = \{(x, f_A(x)) : x \in E\},$$

where $f_A : E \rightarrow P(U)$ is such that $f_A(x) = \emptyset$ if $x \notin A$.

Here, $f_A$ is called the approximate function of the soft set $F_A$, and the value $f_A(x)$ is a set called the $x$-element of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary.

2.2. Example. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ a set of parameters. If $A = \{x_2, x_3, x_4\}$ and $f_A(x_2) = \{u_2, u_4\}$, $f_A(x_3) = \emptyset$, $f_A(x_4) = U$, then the soft-set $F_A$ is written as

$$F_A = \{(x_2, \{u_2, u_4\}), (x_4, U)\},$$

2.3. Definition. [28] Let $U$ be a universe. Then a fuzzy set $X$ over $U$ is a function defined as follows:

$$X = \{(\mu_X(u)/u) : u \in U\},$$

where $\mu_X : U \rightarrow [0, 1]$.

Here, $\mu_X$ is called the membership function of $X$, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. This value represents the degree of $u$ belonging to the fuzzy set $X$.

3. FP-soft sets

In this section, we recall FP-soft sets and their operations [8]. The basic ideas of FP-soft set theory and its extensions, as well as its applications, can be found in [8].

In section 2, subsets of $E$ are classical sets, denoted by the letters $A, B, C, \ldots$, but in this section the subsets of $E$ will be fuzzy and denoted by the letters $X, Y, Z, \ldots$ to avoid confusion and complexity of the symbols.
3.1. Definition. [8] Let \( U \) be an initial universe, \( P(U) \) be the power set of \( U \), \( E \) a set of parameters and \( X \) a fuzzy set over \( E \). Then a FP-soft set \((f_X, E)\) on the universe \( U \) is defined as follows:

\[
(f_X, E) = \{\langle \mu_X(x)/x, f_X(x) \rangle : x \in E \},
\]

where \( \mu_X : E \rightarrow [0,1] \) and \( f_X : E \rightarrow P(U) \) are such that \( f_X(x) = \emptyset \) if \( \mu_X(x) = 0 \).

Here \( f_X \) is called the approximate function and \( \mu_X \) the membership function of the FP-soft set.

Note that the set of all FP-soft sets over \( U \) will be denoted by \( FPS(U) \).

3.2. Definition. [8] Let \( F_X \in FPS(U) \). If \( \mu_X(x) = 0 \) for all \( x \in E \), then \( F_X \) is called the empty FP-soft set, denoted by \( F_\emptyset \).

3.3. Definition. [8] Let \( F_X \in FPS(U) \). If \( \mu_X(x) = 1 \) and \( f_X(x) = U \) for all \( x \in X \), then \( F_X \) is called the X-universal FP-soft set, denoted by \( F_X \).

If \( X = E \), then the X-universal FP-soft set is called the universal FP-soft set, denoted by \( F_U \).

3.4. Example. [8] Let \( U = \{u_1, u_2, u_3, u_4, u_5\} \) be a universal set and \( E = \{x_1, x_2, x_3, x_4\} \) a set of parameters.

If \( X = \{0.2/x_2, 0.5/x_3, 1/x_4\} \) and \( f_X(x_2) = \{u_2, u_4\}, f_X(x_3) = \emptyset, f_X(x_4) = U \), then the FP-soft set \( F_X \) is written as

\[
F_X = \{(0.2/x_2, \{u_2, u_4\}), (0.5/x_3, \emptyset), (1/x_4, U)\}.
\]

If \( Y = \emptyset \), then the FP-soft set \( F_Y \) is the empty soft set. That is, \( F_Y = F_\emptyset \).

If \( Z = \{1/x_1, 1/x_2\} \) and \( f_Z(x_1) = U, f_Z(x_2) = U \), then the FP-soft set \( F_Z \) is the \( Z \)-universal FP-soft set, that is, \( F_Z = F_Z \).

If \( X = E \) and \( f_X(x_i) = U \) for all \( x_i \in E \), \( i = 1, 2, 3, 4 \), then the FP-soft set \( F_X \) is the universal FP-soft set, that is, \( F_X = F_U \).

3.5. Definition. [8] Let \( F_X, F_Y \in FPS(U) \). Then \( F_X \) is a FP-soft set-subset of \( F_Y \), denoted by \( F_X \supseteq F_Y \), if \( \mu_X(x) \leq \mu_Y(x) \) and \( f_X(x) \subseteq f_Y(x) \) for all \( x \in E \).

3.6. Remark. [8] \( F_X \supseteq F_Y \) does not imply that every element of \( F_X \) is an element of \( F_Y \) as in the definition of a classical subset.

For example, assume that \( U = \{u_1, u_2, u_3, u_4\} \) is a universal set of objects and \( E = \{x_1, x_2, x_3\} \) the set of parameters. If \( X = \{0.5/x_1\} \) and \( Y = \{0.9/x_1, 0.1/x_3\} \), and \( F_X = \{(0.5/x_1, \{u_2, u_4\})\}, F_Y = \{(0.9/x_1, \{u_2, u_3, u_4\}), (0.1/x_3, \{u_1, u_2\})\} \), then for all \( x \in E \), \( \mu_X(x) \leq \mu_Y(x) \) and \( f_X(x) \subseteq f_Y(x) \) is valid. Hence \( F_X \supseteq F_Y \). It is clear that \( (0.5/x_1, \{u_2, u_4\}) \in F_X \) but \( (0.5/x_1, \{u_2, u_4\}) \notin F_Y \).

3.7. Definition. [8] Let \( F_X, F_Y \in FPS(U) \). Then \( F_X \) and \( F_Y \) are FP-soft set-equal, written as \( F_X = F_Y \), if and only if \( \mu_X(x) = \mu_Y(x) \) and \( f_X(x) = f_Y(x) \) for all \( x \in E \).

3.8. Definition. [8] Let \( F_X \in FPS(U) \). Then the complement of \( F_X \), denoted by \( F_X^c \), is the FP-soft set defined by the approximate and membership functions

\[
\mu_X^c(x) = 1 - \mu_X(x) \text{ and } f_X^c(x) = U \setminus f_X(x),
\]

respectively.

3.9. Definition. [8] Let \( F_X, F_Y \in FPS(U) \). Then the union of \( F_X \) and \( F_Y \), denoted by \( F_X \cup F_Y \), is defined by

\[
\mu_{X \cup Y}(x) = \max\{\mu_X(x), \mu_Y(x)\} \text{ and } f_{X \cup Y}(x) = f_X(x) \cup f_Y(x), \text{ for all } x \in E
\]
3.10. Definition. [8] Let \( F_X, F_Y \in FPS(U) \). Then the intersection of \( F_X \) and \( F_Y \), denoted by \( F_X \cap F_Y \), is the FP-soft set defined by the approximate and membership functions
\[
\mu_{X \cap Y}(x) = \min\{\mu_X(x), \mu_Y(x)\} \quad \text{and} \quad f_{X \cap Y}(x) = f_X(x) \cap f_Y(x),
\]
respectively.

3.11. Remark. [8] Let \( F_X \in FPS(U) \). If \( F_X \neq F_0 \) or \( F_X \neq F_U \), then \( F_X \cap F_X \neq F_E \) and \( F_X \cap F_X \neq F_0 \).

4. Means of FP-soft sets

In this section, we define logical means, that is the AND-means and OR-means of two FP-soft sets. We then study some of their desired properties.

4.1. Definition. Let \( F_X_1, F_X_2 \in FPS(U) \). Then for \( (p \in \mathbb{R} - \{0\}) \), the AND-mean of \( F_X_1 \) and \( F_X_2 \), denoted by \( (F_X_1 \wedge F_X_2) \), is the FP-soft set defined by the approximate and membership functions,
\[
\mu_{F_X_1 \wedge F_X_2} : E \to [0, 1], \quad \mu_{F_X_1 \wedge F_X_2}(a) = \left(\frac{1}{2} \sum_{i=1}^{2} \mu_{X_i}(a)^p\right)^{\frac{1}{p}},
\]
and
\[
f_{X_1 \wedge X_2} : E \to P(U), \quad f_{X_1 \wedge X_2}(a) = f_{X_1}(a) \cap f_{X_2}(a),
\]
respectively.

4.2. Definition. Let \( F_X_1, F_X_2 \in FPS(U) \). Then for \( p \in \mathbb{R} - \{0\} \) the OR-mean of \( F_X_1 \) and \( F_X_2 \), denoted by \( (F_X_1 \vee F_X_2) \), is the FP-soft set defined by the approximate and membership functions
\[
\mu_{F_X_1 \vee F_X_2} : E \to [0, 1], \quad \mu_{F_X_1 \vee F_X_2}(a) = \left(\frac{1}{2} \sum_{i=1}^{2} \mu_{X_i}(a)^p\right)^{\frac{1}{p}},
\]
and
\[
f_{X_1 \vee X_2} : E \to P(U), \quad f_{X_1 \vee X_2}(a) = f_{X_1}(a) \cup f_{X_2}(a),
\]
respectively.

Note that for \( F_X_1, F_X_2, \ldots, F_X_n \in \mu SS(U) \) the OR-mean of \( F_X_1, F_X_2, \ldots, F_X_n \), denoted by \( (F_X_1 \vee F_X_2 \vee \cdots \vee F_X_n) \), is a FP-soft set defined by the approximate and membership functions
\[
\mu_{F_X_1 \vee F_X_2 \vee \cdots \vee F_X_n} : E \to [0, 1], \quad \mu_{F_X_1 \vee F_X_2 \vee \cdots \vee F_X_n}(a) = \left(\frac{1}{n} \sum_{i=1}^{n} \mu_{X_i}(a)^p\right)^{\frac{1}{p}},
\]
and
\[
f_{X_1 \vee X_2 \vee \cdots \vee X_n} : E \to P(U), \quad f_{X_1 \vee X_2 \vee \cdots \vee X_n}(a) = f_{X_1}(a) \cup \cdots \cup f_{X_n}(a),
\]
respectively. Similarly, \( F_X_1 \wedge F_X_2 \wedge \cdots \wedge F_X_2 \) can easily be defined.

Note that the symbols \( \wedge \) and \( \vee \) used in the subscripts of the approximate and membership functions are not operations on fuzzy sets. They indicate that \( f_{X_1 \wedge X_2} \) and \( f_{X_1 \vee X_2} \) are the approximate functions of \( F_X_1 \wedge F_X_2 \) and \( F_X_1 \vee F_X_2 \), respectively, and that \( \mu_{X_1 \wedge X_2} \) and \( \mu_{X_1 \vee X_2} \) are the membership functions of \( F_X_1 \wedge F_X_2 \) and \( F_X_1 \vee F_X_2 \), respectively.

4.3. Example. Let \( p=1 \). Assume that \( U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} \) is a universal set and \( E = \{a_1, a_2, a_3, a_4, a_5\} \) a set of parameters. If \( X_1 = \{0.5/a_2, 0.2/a_3, 1/a_4\} \) and
$X_2 = \{0.9/a_1, 0.3/a_3, 0.4/a_4, 0.8/a_5\}$ are two fuzzy sets over $E$, then we can write the following FP-soft sets,

$F_{X_1} = \{(0.5/a_2, \{u_2, u_3, u_4, u_7\}), (0.2/a_3, \{u_1, u_2, u_3, u_4\}), (1/a_4, \{u_1, u_2, u_5, u_7, u_8\})\}$

\[
\begin{array}{c|cccc}
F_{X_1} & 0.25/a_2 & 0.25/a_3 & 0.70/a_4 \\
\hline 
u_1 & 0 & 1 & 1 \\
u_2 & 1 & 1 & 1 \\
u_3 & 1 & 1 & 0 \\
u_4 & 1 & 1 & 0 \\
u_5 & 0 & 0 & 1 \\
u_6 & 0 & 0 & 0 \\
u_7 & 1 & 0 & 1 \\
u_8 & 0 & 0 & 1 \\
\end{array}
\]

$F_{X_2} = \{(0.9/a_1, \{u_1, u_2, u_5, u_6\}), (0.3/a_3, \{u_3, u_4, u_5, u_8\}), (0.4/a_4, U), (0.8/a_5, \{u_3, u_4, u_6, u_7, u_8\})\}$

\[
\begin{array}{c|cccc}
F_{X_2} & 0.9/a_1 & 0.3/a_3 & 0.4/a_4 & 0.8/a_5 \\
\hline 
u_1 & 1 & 0 & 1 & 0 \\
u_2 & 1 & 0 & 1 & 0 \\
u_3 & 0 & 1 & 1 & 1 \\
u_4 & 0 & 1 & 1 & 1 \\
u_5 & 1 & 1 & 1 & 0 \\
u_6 & 1 & 0 & 1 & 1 \\
u_7 & 0 & 0 & 1 & 1 \\
u_8 & 0 & 1 & 1 & 1 \\
\end{array}
\]

$F_{X_1} \vee F_{X_2} = \{(0.45/a_1, \{u_1, u_2, u_5, u_6\}), (0.25/a_2, \{u_2, u_3, u_4, u_5, u_7\}), (0.25/a_3, \{u_1, u_2, u_3, u_4, u_5, u_8\}), (0.70/a_4, U), (0.4/a_5, \{u_3, u_4, u_6, u_7, u_8\})\}$

\[
\begin{array}{c|cccc}
F_{X_1} \vee F_{X_2} & 0.25/a_2 & 0.25/a_3 & 0.70/a_4 & 0.4/a_5 \\
\hline 
u_1 & 1 & 0 & 1 & 0 \\
u_2 & 1 & 1 & 1 & 0 \\
u_3 & 0 & 1 & 1 & 1 \\
u_4 & 0 & 1 & 1 & 1 \\
u_5 & 1 & 1 & 1 & 0 \\
u_6 & 1 & 0 & 0 & 1 \\
u_7 & 0 & 1 & 0 & 1 \\
u_8 & 0 & 0 & 1 & 1 \\
\end{array}
\]

Similarly $F_{X_1} \wedge F_{X_2}$ easily can be found.

**4.4. Proposition.** Let $F_{X_1}, F_{X_2} \in FPS(U)$. Then
Proof. For all \( a \in E \), since \( \mu_{X_1}(a) = 1 - \mu_{X_1}(a) \) and \( f_{X_1}(a) = U \setminus f_{X_1}(a) \);

i. \( \mu_{(X_1 \setminus \cdot)}(a) = 1 - \mu_{X_1}(a) = 1 - (1 - \mu_{X_1}(a)) = \mu_{X_1}(a) \)

and

\[ f_{(X_1 \setminus \cdot)}(a) = U \setminus (U \setminus f_{X_1}(a)) = f_{X_1}(a). \]

Therefore we get

\[ (F_{X_1}^\mu) = F_{X_1}. \]

ii. \( F_{\emptyset}^\mu = F_{\emptyset} \) can be shown similarly. \( \square \)

4.5. Proposition. Let \( F_X, F_Y, F_Z \in FPS(U) \). Then

i. \( F_X \wedge F_X = F_{X_1} \wedge F_{X_2} \iff F_{X_2} = F_{X_3} \)

ii. \( F_X \vee F_X = F_{X_1} \vee F_{X_3} \iff F_{X_2} = F_{X_3} \).

Proof. For all \( a \in E \),

i. \( F_X \wedge F_X = F_{X_1} \wedge F_{X_2} \iff \left( \frac{1}{2} \right)^{\frac{1}{p}} (\mu_{X_1}(a)^p + \mu_{X_2}(a)^p) \right)^\frac{1}{p} \)

\[ = \left( \frac{1}{2} \right)^{\frac{1}{p}} (\mu_{X_1}(a)^p + \mu_{X_2}(a)^p) \right)^\frac{1}{p} \]

\[ \iff \mu_{X_2}(a) = \mu_{X_3}(a) \]

and

\[ f_{X_1}(a) \cap f_{X_2}(a) = f_{X_1}(a) \cap f_{X_3}(a) \iff f_{X_2}(a) = f_{X_3}(a). \]

Hence we obtain

\( F_{X_1} \wedge F_{X_2} = F_{X_1} \wedge F_{X_3} \iff F_{X_2} = F_{X_3}. \)

ii. \( F_X \vee F_X = F_{X_1} \vee F_{X_3} \iff \left( \frac{1}{2} \right)^{\frac{1}{p}} (\mu_{X_1}(a)^p + \mu_{X_3}(a)^p) \right)^\frac{1}{p} \)

\[ = \left( \frac{1}{2} \right)^{\frac{1}{p}} (\mu_{X_1}(a)^p + \mu_{X_3}(a)^p) \right)^\frac{1}{p} \]

\[ \iff \mu_{X_2}(a) = \mu_{X_3}(a) \]

and

\[ f_{X_1}(a) \cup f_{X_2}(a) = f_{X_1}(a) \cup f_{X_3}(a) \iff f_{X_2}(a) = f_{X_3}(a). \]

Therefore we obtain

\( F_{X_1} \vee F_{X_2} = F_{X_1} \vee F_{X_3} \iff F_{X_2} = F_{X_3}. \) \( \square \)

4.6. Proposition. Let \( F_X, F_Y, F_Z \in FPS(U) \). Then

i. \( F_X \vee F_X = F_{X_1} \)

ii. \( F_X \wedge F_X = F_{X_1} \)

Proof. Follows easily from Definition 4.1 and Definition 4.2. \( \square \)

4.7. Proposition. Let \( F_X, F_X, F_X \in FPS(U) \). Then

i. \( (F_X \vee F_X) \vee F_X = F_{X_1} \vee (F_{X_2} \vee F_{X_3}) \),
ii. \((F_X \wedge F_{X'} \wedge F_{X''}) = F_X \wedge (F_{X'} \wedge F_{X''})\).

**Proof.** Follows easily from Definition 4.1 and Definition 4.2. □

4.8. **Proposition.** Let \(F_{X_1}, F_{X_2}, F_{X_3} \in \text{FPS}(U)\). Then

i. \(F_{X_1} \wedge (F_{X_2} \wedge F_{X_3}) = (F_{X_1} \wedge F_{X_2}) \wedge F_{X_3}\)

ii. \(F_{X_1} \wedge (F_{X_2} \cup F_{X_3}) = (F_{X_1} \wedge F_{X_2}) \cup (F_{X_1} \wedge F_{X_3})\)

iii. \(F_{X_1} \wedge (F_{X_2} \cap F_{X_3}) = (F_{X_1} \wedge F_{X_2}) \cap (F_{X_1} \wedge F_{X_3})\)

iv. \(F_{X_1} \wedge (F_{X_2} \cup F_{X_3}) = (F_{X_1} \wedge F_{X_2}) \cup (F_{X_1} \wedge F_{X_3})\)

**Proof.**

i. \(F_{X_1} \wedge (F_{X_2} \cap F_{X_3}) = \left(\frac{1}{2}\right) \cdot \left(\mu_{X_1}(a) + \min \{\mu_{X_2}(a), \mu_{X_3}(a)\}\right)^{1/2}\)

\[\left(\frac{1}{2}\right) \cdot \left(\mu_{X_1}(a) + \min \{\mu_{X_2}(a), \mu_{X_3}(a)\}\right)^{1/2}\]

so we get \(F_{X_1} \wedge F_{X_2} \cap F_{X_3}\).

ii. Similarly, \(F_{X_1} \wedge (F_{X_2} \cup F_{X_3}) = (F_{X_1} \wedge F_{X_2}) \cup (F_{X_1} \wedge F_{X_3})\) easily follows.

iii. \(F_{X_1} \wedge (F_{X_2} \cap F_{X_3}) = \left(\frac{1}{2}\right) \cdot \left(\mu_{X_1}(a) + \min \{\mu_{X_2}(a), \mu_{X_3}(a)\}\right)^{1/2}\)

\[\left(\frac{1}{2}\right) \cdot \left(\mu_{X_1}(a) + \min \{\mu_{X_2}(a), \mu_{X_3}(a)\}\right)^{1/2}\]

so we obtain \((F_{X_1} \wedge F_{X_2}) \cap (F_{X_1} \wedge F_{X_3})\).

iv. Similarly, \(F_{X_1} \wedge (F_{X_2} \cup F_{X_3}) = (F_{X_1} \wedge F_{X_2}) \cup (F_{X_1} \wedge F_{X_3})\) can easily be proved. □

4.9. **Proposition.** Let \(F_{X_1}, F_{X_2}, F_{X_3} \in \text{FPS}(U)\). Then

i. \(F_{X_1} \vee F_{X_2} = F_{X_2} \vee F_{X_1}\)

ii. \(F_{X_1} \wedge F_{X_2} = F_{X_2} \wedge F_{X_1}\)

**Proof.** For all \(a \in E\),

i. \(F_{X_1} \vee F_{X_2} = \left(\frac{1}{2}\right) \cdot \left(\mu_{X_1}(a) + \mu_{X_2}(a)\right)^{1/2}\)

\[\left(\frac{1}{2}\right) \cdot \left(\mu_{X_1}(a) + \mu_{X_2}(a)\right)^{1/2}\]

and \(f_{X_1}(a) \cap f_{X_2}(a) = f_{X_2}(a) \cap f_{X_1}(a)\).

Therefore, we obtain \(F_{X_1} \vee F_{X_2} = F_{X_2} \vee F_{X_1}\).

ii. \(F_{X_1} \wedge F_{X_2} = \left(\frac{1}{2}\right) \cdot \left(\mu_{X_1}(a) + \mu_{X_2}(a)\right)^{1/2}\)

\[\left(\frac{1}{2}\right) \cdot \left(\mu_{X_1}(a) + \mu_{X_2}(a)\right)^{1/2}\]
and

\[ f_{X_1}(a) \cup f_{X_2}(a) = f_{X_2}(a) \cup f_{X_1}(a), \]

therefore we get

\[ F_{X_1} \wedge F_{X_2} = F_{X_2} \wedge F_{X_1}. \]

\[ \square \]

4.10. Proposition. Let \( F_{X_1}, F_{X_2}, F_{X_3} \in FPS(U) \). Then

i. \( F_{X_1} \wedge (F_{X_2} \vee F_{X_3}) = (F_{X_1} \wedge F_{X_2}) \vee (F_{X_1} \wedge F_{X_3}) \),

ii. \( F_{X_1} \vee (F_{X_2} \wedge F_{X_3}) = (F_{X_1} \vee F_{X_2}) \wedge (F_{X_1} \vee F_{X_3}) \).

Proof. Follows easily from Definition 4.1 and Definition 4.2. \( \square \)

5. Soft fuzzification operators

In this section, we give soft fuzzification operators by using the AND-means, OR-means that convert two FP-soft sets to a fuzzy set.

5.1. Definition. In this definition, for the shake of brevity, we use \( T \) instead of \( F_{X_1} \wedge F_{X_2} \). Let \( f_T \) be an approximation function and \( \mu_T \) a membership function of \( T \). Then, an AND-soft fuzzification operator, denoted by \( s_T \wedge \) is defined by

\[ s_T \wedge T = \{ \mu_T(u) / u \in f_T(a), \mu_T(u) \in [0,1] \} \]

where

\[ \mu_T(u) = \frac{1}{|U|} \sum_{i,j} \mu_T(a_i) \chi_{T}(a_j)(u_j) \]

and where

\[ \chi_{T}(a_j)(u_j) = \begin{cases} 1, & u_j \in f_T(a_i), \\ 0, & u_j \notin f_T(a_i). \end{cases} \]

The set \( s_T \wedge T \) is a fuzzy set called an OR-decision fuzzy set over \( U \).

Note that \( s_T \vee (F_{X_1} \wedge F_{X_2}) \) is defined in a similar way.

5.2. Proposition. Let \( F_{X_1}, F_{X_2} \in FPS(U) \). Then

i. \( s_T \wedge (F_{X_1} \wedge F_{X_2}) = s_T \wedge (F_{X_2} \wedge F_{X_1}) \),

ii. \( s_T \vee (F_{X_1} \wedge F_{X_2}) = s_T \vee (F_{X_2} \wedge F_{X_1}) \).

Proof.

i. \( F_{X_1} \wedge F_{X_2} = F_{X_2} \wedge F_{X_1} \implies s_T \wedge (F_{X_1} \wedge F_{X_2}) = s_T \wedge (F_{X_2} \wedge F_{X_1}) \).

ii. \( F_{X_1} \vee F_{X_2} = F_{X_2} \vee F_{X_1} \implies s_T \vee (F_{X_1} \vee F_{X_2}) = s_T \vee (F_{X_2} \vee F_{X_1}) \).

\( \square \)

6. FP-soft set-decision making methods

In this section we construct FP-soft set-decision making methods. Assume that \( U \) is an initial universe that contains alternatives and that \( E \) is a set of parameters. We then construct an OR-FP-soft set decision making method by the following algorithm to produce a decision fuzzy set from \( U \):

i. Choose feasible fuzzy subsets \( X_1 \) and \( X_2 \) over \( E \);
ii. Construct the FP-soft sets \( F_{X_1} \) and \( F_{X_2} \) over \( U \);
iii. Find the OR-means \( F_{X_1} \vee F_{X_2} \);
iv. Compute the OR-decision fuzzy set \( s_0(F_{X_1} \vee F_{X_2}) \).

Note that, in a similar way, we can construct an AND-FP-soft set-decision making method. Now, we can give an example for the FP-soft set-decision making method.
6.1. Example. Assume that a country has a set of different places for industrial plants:

\[ U = \{ u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8 \}. \]

For \( i = 1, 2, 3, 4, 5, 6, 7, 8 \) the parameters \( u_i \) will stand for “Istanbul”, “Denizli”, “Ankara”, “Izmir”, “Adana”, “Mersin”, “Antalya” and “Bursa” respectively, which may be characterized by a set of parameters \( E = \{ a_1, a_2, a_3, a_4, a_5 \}. \) For \( i = 1, 2, 3, 4, 5 \) the parameters \( a_i \) stand for “transportation”, “productive power”, “marketing”, “profit” and “sale” respectively.

Let \( p=1 \), and suppose that a company planning to build a plant in that country sends two administrators Mr X and Mr Y to the country to decide the best place to build the plant. If each member is to consider his own fuzzy set of parameters, then we can select a place for the plant on the basis of the sets of members’ parameters by using the Or-means-FP-soft set-decision making method as follows.

**Step i** : Mr X and Mr Y construct fuzzy sets, say

\[
X_1 = \{ 0.9/a_1, 0.1/a_2, 0.5/a_3, 0.9/a_5 \}, \text{ and } X_2 = \{ 0.2/a_1, 0.3/a_3, 0.7/a_4, 0.8/a_5 \},
\]

over \( E \), respectively.

**Step ii** : \( F_{X_1} \) and \( F_{X_2} \) over \( U \) are written by

\[
F_{X_1} = \left\{ (0.9/a_2, \{ u_3, u_4, u_5 \}), (0.1/a_3, \{ u_1, u_2, u_3, u_4 \}), (0.5/a_4, \{ u_5, u_7, u_8 \}), (0.9/a_5, \{ u_3, u_4, u_5 \}) \right\}, \\
F_{X_2} = \left\{ (0.2/a_1, \{ u_1, u_2, u_5, u_6 \}), (0.3/a_3, \{ u_3, u_4, u_5, u_8 \}), (0.7/a_4, \{ u_5, u_8 \}), (0.8/a_5, \{ u_3, u_4, u_6, u_7, u_9 \}) \right\}.
\]

**Step iii** : Calculate the OR-means of \( F_{X_1} \) and \( F_{X_2} \) as

\[
F_{X_1} \bigvee F_{X_2} = \left\{ (0.10/a_1, \{ u_1, u_2, u_5, u_6 \}), (0.45/a_2, \{ u_3, u_4, u_5 \}), (0.20/a_3, \{ u_1, u_2, u_3, u_4, u_5, u_8 \}), (0.60/a_4, \{ u_5, u_7, u_8 \}), (0.85/a_5, \{ u_3, u_4, u_5, u_6, u_7, u_8 \}) \right\}
\]

**Step vi** : The OR-decision fuzzy set \( s_0(F_{X_1} \bigvee F_{X_2}) \) is computed as

\[
\{ 0.037/u_1, 0.037/u_2, 0.187/u_3, 0.187/u_4, 0.275/u_5, 0.118/u_6, 0.181/u_7, 0.206/u_8 \}
\]

where the alternative \( u_5 \) is the decision-maker’s best choice since it has the largest degree of membership, 0.2750, among the others.

7. Conclusion

Molodtsov, in 1999, proposed soft set theory as a general mathematical tool for dealing with the problems that contain uncertainties. In this paper, after giving most of the fundamental definitions of FP-soft set theory needed to develop applications of soft set theory we defined means in FP-soft theory and studied their properties. We then defined decision making methods in FP-soft set theory. We also give an application which shows that they can be successfully applied to problems that contain uncertainties. In the future, these results can also be applied similarly using other theories such as fuzzy sets and rough sets.

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References


