

A FRACTIONAL ORDER SEIR MODEL WITH DENSITY DEPENDENT DEATH RATE

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Abstract

In this paper, we introduce a fractional order SEIR epidemic model with vertical transmission, where the death rate of the population is density dependent, i.e., dependent on the population size. It is also assumed that there exists an infection related death rate. We show the existence of nonnegative solutions of the model, and also give a detailed stability analysis of disease free and positive fixed points. A numerical example is also presented.

Keywords: Fractional derivative, Initial value problem, SEIR model, Stability, Numerical solution.

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1. Introduction

Mathematical modeling in epidemiology provides new aspects in understanding the spread of diseases, and it suggests control strategies [3]. One of the early models in epidemiology was introduced in 1927 [9] to predict the spreading behaviour of a disease. Since then, many epidemic models have been derived [8]. In [7] a detailed analysis for integer order SEIR models with vertical transmission within a constant population can be found. There are also several papers [12, 13] about epidemic models within a nonconstant population, which is more realistic.

Although a large number of works has been done on modeling the dynamics of epidemiological diseases, it has been restricted to integer order (delay) differential equations. In recent years, it has turned out that many phenomena in different fields can be described very successfully by the models using fractional order differential equations [1,2,4,6].

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