BITOPOLOGICAL S-CLUSTER SETS

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Abstract

A new type of cluster set called an $S$-cluster set of functions and multifunctions between bitopological spaces has been introduced. Expressions and conditions for the degeneracy of such sets are also found. As an application, characterizations of Hausdorff and $S$-closed bitopological spaces are achieved via such cluster sets.

Keywords:  Bitopological spaces, $ij$-$S$-cluster sets, $ij$-semi open sets, $ij$-$S$-closed spaces.

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1. Introduction

The theory of cluster sets has been studied and applied for a long time in real and complex analysis. But cluster sets for functions between topological spaces was first initiated by Wetson [15], and many other topologists like Joseph [4] and Hamlett [3] have made valuable contributions in the area. In bitopological spaces, Nandi and Mukherjee [11] introduced and studied a type of cluster set called a $\delta$-cluster set. In this paper, our intention is to extend a type of cluster set of functions and multifunctions called an $S$-cluster set [10], to the bitopological setting. Using these cluster sets, we give a new characterization of Hausdorff and $S$-closed bitopological spaces.

Throughout the paper, $(X, \tau_1, \tau_2)$ and $(Y, \sigma_1, \sigma_2)$ (or briefly, $X$ and $Y$) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let $A$ be a subset of $X$. By $i$-$\text{int}(A)$ and $i$-$\text{cl}(A)$, we denote respectively the interior and closure of $A$ with respect to $\tau_i$ (or $\sigma_i$) for $i = 1, 2$. Also, $i, j = 1, 2$ and $i \neq j$.

A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called $ij$-semiopen [2] if there exists $U \in \tau_i$ such that $U \subseteq A \subseteq j$-$\text{cl}(U)$. Equivalently, $A$ is $ij$-semiopen if and only if $A \subseteq j$-$\text{cl}(i$-$\text{int}(A))$. The complement of an $ij$-semiopen set is called $ij$-semiclosed. The family of all $ij$-semiopen (resp. $\tau_i$-open) sets of $X$ containing a given subset $A$ of $X$ is denoted by $ij$-$SO(A)$ (resp. $\tau_i(A)$). If $A = \{x\}$, we write $ij$-$SO(x)$ and $\tau_i(x)$ instead of $ij$-$SO(\{x\})$ and $\tau_i(\{x\})$, respectively.

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