Erratum and Notes for Near Groups on Nearness Approximation Spaces

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The authors would like to write some notes and correct errors in the original publication of the article [1]. The notes are given below:

0.1. Remark. In page 550, in Definition 3.1., (1) and (2) properties have to hold in $N_r(B)^*G$. Sometimes they may be hold in $O\setminus N_r(B)^*G$, then $G$ is not a near group on nearness approximation space.

Example 3.3. and 3.4. are nice examples of this case. In Example 3.3., if we consider associative property $(b \cdot e) \cdot b = b \cdot (e \cdot b)$ for $b, e \in H \subset G$, we obtain $i = i$, but $i \in O\setminus N_r(B)^*H$. Hence, we can observe that if the associative property holds in $O\setminus N_r(B)^*H$, then $H$ can not be a subnear group of near group $G$. Consequently, Example 3.3. and 3.4. are incorrect, i.e., they are not subnear groups of near group $G$.

0.2. Remark. Multiplying of finite number of elements in $G$ may not always belongs to $N_r(B)^*G$. Therefore always we cannot say that $x^n \in N_r(B)^*G$, for all $x \in G$ and some positive integer $n$. If $(N_r(B)^*G, \cdot)$ is groupoid, then we can say that $x^n \in N_r(B)^*G$, for all $x \in R$ and all positive integer $n$.

In Example 3.2., the properties (1) and (2) hold in $N_r(B)^*G$. Hence $G$ is a near group on nearness approximation space.

The corrections are given below:

In page 548, in subsection 2.4.1., definition of $B$-lower approximation of $X \subseteq O$ must be

$$B_*X = \bigcup_{[x]_B \subseteq X} [x]_B.$$
In page 554, Theorem 3.8. must be as in Theorem 0.3:

0.3. Theorem. Let \( G \) be a near group on nearness approximation space, \( H \) a nonempty subset of \( G \) and \( N_r(B)^*H \) a groupoid. \( H \subseteq G \) is a subnear group of \( G \) if and only if \( x^{-1} \in H \) for all \( x \in H \).

Proof. Suppose that \( H \) is a subnear group of \( G \). Then \( H \) is a near group and so \( x^{-1} \in H \) for all \( x \in H \). Conversely, suppose \( x^{-1} \in H \) for all \( x \in H \). By the hypothesis, since \( N_r(B)^*H \) is a groupoid and \( H \subseteq G \), then closure and associative properties hold in \( N_r(B)^*H \). Also we have \( x \cdot x^{-1} = e \in N_r(B)^*H \). Hence \( H \) is a subnear group of \( G \). \( \square \)

In page 554, Theorem 3.9. must be as in Theorem 0.4:

0.4. Theorem. Let \( H_1 \) and \( H_2 \) be two near subgroups of the near group \( G \) and \( N_r(B)^*H_1, N_r(B)^*H_2 \) groupoids. If

\[ (N_r(B)^*H_1) \cap (N_r(B)^*H_2) = N_r(B)^*(H_1 \cap H_2), \]

then \( H_1 \cap H_2 \) is a near subgroup of near group \( G \).

Proof. Suppose \( H_1 \) and \( H_2 \) be two near subgroups of the near group \( G \). It is obvious that \( H_1 \cap H_2 \subseteq G \). Since \( N_r(B)^*H_1, N_r(B)^*H_2 \) are groupoids and \( (N_r(B)^*H_1) \cap (N_r(B)^*H_2) = N_r(B)^*(H_1 \cap H_2) \), \( N_r(B)^*(H_1 \cap H_2) \) is a groupoid. Consider \( x \in H_1 \cap H_2 \). Since \( H_1 \) and \( H_2 \) are near subgroups, we have \( x^{-1} \in H_1 \) and \( x^{-1} \in H_2 \), i.e., \( x^{-1} \in H_1 \cap H_2 \). Thus from Theorem 0.3 \( H_1 \cap H_2 \) is a near subgroup of \( G \). \( \square \)

In page 555, proof of Theorem 5.3. has some typos. It must be as in Theorem 0.5:

0.5. Theorem. Let \( G \) be a near group on nearness approximation space and \( N \) a subnear group of \( G \). \( N \) is a subnear normal group of \( G \) if and only if \( a \cdot n \cdot a^{-1} \in N \) for all \( a \in G \) and \( n \in N \).

Proof. Suppose \( N \) is a near normal subgroup of near group \( G \). We have \( a \cdot N \cdot a^{-1} = N \) for all \( a \in G \). For any \( n \in N \), therefore we have \( a \cdot n \cdot a^{-1} \in N \). Suppose \( N \) is a near subgroup of near group \( G \). Suppose \( a \cdot n \cdot a^{-1} \in N \) for all \( a \in G \) and \( n \in N \). We have \( a \cdot N \cdot a^{-1} \subseteq N \). Since \( a^{-1} \in G \), we get \( a \cdot (a^{-1} \cdot N \cdot a) \cdot a^{-1} \subseteq a \cdot N \cdot a^{-1} \), i.e., \( N \subseteq a \cdot N \cdot a^{-1} \). Since \( a \cdot N \cdot a^{-1} \subseteq N \) and \( N \subseteq a \cdot N \cdot a^{-1} \), we obtain \( a \cdot N \cdot a^{-1} = N \). Therefore \( N \) is a subnear normal group of \( G \). \( \square \)

In page 556, Theorem 6.6. must be as in Theorem 0.6:

0.6. Theorem. Let \( G_1 \subset O_1,G_2 \subset O_2 \) be near groups that are near homomorphic, \( N \) near homomorphism kernel and \( N_r(B)^*N \) a groupoid. Then \( N \) is a near normal subgroup of \( G_1 \).

In page 557, Theorem 6.7. must be as in Theorem 0.7:

0.7. Theorem. Let \( G_1 \subset O_1,G_2 \subset O_2 \) be near homomorphic groups, \( H_1 \) and \( N_1 \) a near subgroup and a near normal subgroup of \( G_1 \), respectively and \( N_r(B)^*H_1 \) groupoid. Then we have the following.
(1) If $\varphi (N_{r_1} (B)^* H_1) = N_{r_2} (B)^* \varphi (H_1)$, then $\varphi (H_1)$ is a near subgroup of $G_2$.

(2) if $\varphi (G_1) = G_2$ and $\varphi (N_{r_1} (B)^* N_1) = N_{r_2} (B)^* \varphi (N_1)$, then $\varphi (N_1)$ is a near normal subgroup of $G_2$.

In page 557, Theorem 6.8. must be as in Theorem 0.8:

**0.8. Theorem.** Let $G_1 \subset O_1$, $G_2 \subset O_2$ be near homomorphic groups, $H_2$ and $N_2$ a near subgroup and a near normal subgroup of $G_2$, respectively and $N_{r_1} (B)^* H_1$ groupoid. Then we have the following.

(1) if $\varphi (N_{r_1} (B)^* H_1) = N_{r_2} (B)^* H_2$, then $H_1$ is a near subgroup of $G_1$ where $H_1$ is the inverse image of $H_2$.

(2) if $\varphi (G_1) = G_2$ and $\varphi (N_{r_1} (B)^* N_1) = N_{r_2} (B)^* N_2$, then $N_1$ is a near normal subgroup of $G_1$ where $N_1$ is the inverse image of $N_2$.

We apologize to the readers for any inconvenience of these errors might have caused.

References