

# TOPOLOGICAL K-THEORY OF THE CLASSIFYING SPACES OF CYCLIC AND DIHEDRAL GROUPS

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## Abstract

We make a little survey and also present some new results on the topological K-theory of the classifying spaces of cyclic and dihedral groups.

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## 1. Introduction

The  $K$ -ring of a  $CW$ -complex  $X$ , denoted by  $K(X)$ , is defined by the ring completion of the semi-ring of the isomorphism classes of complex vector bundles over  $X$ . The  $KO$ -ring of  $X$ , denoted by  $KO(X)$ , is defined similarly, by means of real vector bundles over  $X$ . Similar rings can be constructed for the other fields like the field of quaternions or the finite fields. One of the most interesting questions of the topological  $K$ -theory is to determine these rings when  $X$  is the classifying space  $BG$  of a group, in particular a finite group,  $G$ . See, for example, the description given in [5] for the  $KO$ -ring of the skeletons of the classifying space of the cyclic group of order  $2^n$ .

In this note, we will make a brief survey and also present some new results for the  $K$ -rings and  $KO$ -rings of the classifying space of the cyclic and the dihedral groups.

Before starting the presentation, we should mention two important theorems in topological  $K$ -theory. Firstly, there is the Atiyah-Segal completion theorem (ASCT) which states that  $K(BG)$  is isomorphic to the completion of the complex representation ring of  $G$  at the augmentation ideal, that is,

$$K(BG) = R(G)_{\hat{I}}$$

Basically, this theorem says that  $K(BG)$  is another way of writing the elements of  $R(G)$ , as formal sums of their reductions and what we are doing here is not more than representation theory with a little geometry added. A similar theorem holds for  $KO$ -rings and the real representation ring  $RO(G)$ .

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