Hacettepe Journal of Mathematics and Statistics
Volume 42 (4) (2013), 381 – 385

TOPOLOGICAL K-THEORY OF THE CLASSIFYING SPACES OF CYCLIC AND DIHEDRAL GROUPS

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Received 28 : 06 : 2012 : Accepted 01 : 11 : 2012

Abstract

We make a little survey and also present some new results on the topological K-theory of the classfying spaces of cyclic and dihedral groups.

Keywords: K-Theory, Representation Theory, Classifying Space

2000 AMS Classification: 55R50; 20C10

1. Introduction

The K-ring of a CW-complex X, denoted by $K(X)$, is defined by the ring completion of the semi-ring of the isomorphism classes of complex vector bundles over X. The KO-ring of X, denoted by $KO(X)$, is defined similarly, by means of real vector bundles over X. Similar rings can be constructed for the other fields like the field of quaternions or the finite fields. One of the most interesting questions of the topological K-theory is to determine these rings when X is the classifying space BG of a group, in particular a finite group, G. See, for example, the description given in [5] for the KO-ring of the skeletons of the classifying space of the cyclic group of order 2^n.

In this note, we will make a brief survey and also present some new results for the K-rings and KO-rings of the classifying space of the cyclic and the dihedral groups.

Before starting the presentation, we should mention two important theorems in topological K-theory. Firstly, there is the Atiyah-Segal completion theorem (ASCT) which states that $K(BG)$ is isomorphic to the completion of the complex representation ring of G at the augmentation ideal, that is,

$$K(BG) = R(G)\hat{}_I$$

Basically, this theorem says that $K(BG)$ is another way of writing the elements of $R(G)$, as formal sums of their reductions and what we are doing here is not more than representation theory with a little geometry added. A similar theorem holds for KO-rings and the real representation ring $RO(G)$.

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