ON THE WEAK CONVERGENCE OF
THE ERGODIC DISTRIBUTION FOR AN
INVENTORY MODEL OF TYPE (s,S)

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Abstract
In this study, a renewal - reward process with a discrete interference of chance is constructed. This process describes in particular a semi-Markovian inventory model of type (s,S). The ergodic distribution of this process is expressed by a renewal function, and a second-order approximation for the ergodic distribution of the process is obtained as $S - s \to \infty$ when the interference has a triangular distribution. Then, the weak convergence theorem is proved for the ergodic distribution and the limit distribution is derived. Finally, the accuracy of the approximation formula is tested by the Monte Carlo simulation method.

Keywords: Renewal-reward process, Discrete interference of chance, Asymptotic expansion, Triangular distribution, Weak convergence, Renewal function.


1. Introduction
Many interesting problems that are related to the theories of inventory, stock control, queuing, reliability, mathematical biology, stochastic finance, mathematical insurance, etc., can be expressed by renewal-reward or random walk processes. There are many interesting studies on these topics in the literature, see for example [1, 2, 4, 6, 9, 15, 16, 17, 18, 19, 20, 21]. But most of these studies are generally theoretical and are not helpful enough in solving concrete problems in practice due to the complexity of their mathematical structure. In addition to these theoretical studies, there are also some

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