On fuzzy soft semi-pre-open sets and fuzzy soft semi-pre-continuous mappings

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Abstract

The main aim of this paper is to initiate and explore the properties of fuzzy soft semi-pre-open(closed) sets. We introduce and investigate fuzzy soft semi-pre-interior and fuzzy soft semi-pre-closure in fuzzy soft topological spaces. Moreover, we define and study the characterizations of fuzzy soft semi-pre-continuous and fuzzy soft semi-pre-open(closed) mappings in fuzzy soft topological spaces.

Keywords: Fuzzy soft sets, Fuzzy soft topology, Fuzzy soft semi-pre-open(closed), Fuzzy soft semi-pre-interior(closure), Fuzzy soft semi-pre-continuous, Fuzzy soft semi-pre-open(closed) mappings.

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1. Introduction

D. Molodtsov [31] presented the concept of soft set as a new mathematical tool to solve varieties of complicated problems having uncertainties in real life, economics, engineering and computer sciences, social and medical sciences etc. In [32], D. Molodtsov et. al discussed the applications of soft sets in different fields. P. K. Maji et. al [27-28] explored the fundamental concepts of soft set theory in detail and established the applications of soft sets in decision making problems.

F. Feng et.al [12] introduced the soft product in soft set theory. Moreover, they gave generalization of uni-int decision making scheme in [13]. The relation of soft sets in information systems, criteria of measuring the sound quality, normal parameter reduction and classification of natural texture have been studied in [35],[26],[39], [25] and [33]. M. Shabir and M. Naz [36] defined and discussed the basic concepts of soft topological spaces. Later on, many researchers [1], [3-4],[9-11],[14],[17-20],[30],[41] explored different algebraic structures of soft topological spaces.

B. Chen [6-7] defined and discussed soft semi-open(closed) sets in soft topological spaces.

S. Hussain [21] continued to study the algebraic structures of soft semi-open(closed) sets.

L. A. Zadeh [40] initiated the concept of fuzzy soft sets and provided a natural base to

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handle and improve mathematically the fuzzy phenomenon found in different areas of knowledge. C. L. Chang [5] studied and discussed fuzzy topological spaces. B. Ahmad and A. Kharral [2] investigated fuzzy soft sets and established fuzzy soft functions on fuzzy soft classes. The work on fuzzy soft topology improved further in [15-16],[24],[29], [34],[37-38].

In 2015, S. Hussain [22], initiated and explored the fuzzy soft semi-open(closed) sets by combining fuzzy soft sets and soft semi-open sets. S. Hussain investigated the properties of fuzzy soft semi-interior(closure), fuzzy soft semi-exterior, fuzzy soft semi-boundary, fuzzy soft semi-continuous and fuzzy soft semi-open(closed) mapping. Recently in 2016, S. Hussain [23] introduced and examined the basic properties of fuzzy soft α-open(closed) sets, fuzzy soft pre-open(closed) sets, fuzzy soft regular-open(closed) sets and fuzzy soft neighborhood at fuzzy soft point. Moreover, S. Hussain analyzed the relationship between these notions.

2. Preliminaries

First we recall some definitions and results which will use in the sequel.

**Definition 2.1**[40]. A fuzzy set \( f \) on \( X \) is a mapping \( f : X \rightarrow I = [0,1] \). The value \( f(x) \) represents the degree of membership of \( x \in X \) in the fuzzy set \( f \), for \( x \in X \).

**Definition 2.2**[31]. Let \( X \) be an initial universe and \( E \) be a set of parameters. Let \( P(X) \) denote the power set of \( X \) and \( A \) be a non-empty subset of \( E \). A pair \((F,A)\) is called a soft set over \( X \), where \( F \) is a mapping given by \( F : A \rightarrow P(X) \). In other words, a soft set over \( X \) is a parameterized family of subsets of the universe \( X \). For \( e \in A \), \( F(e) \) may be considered as the set of \( e \)-approximate elements of the soft set \((F,A)\).

**Definition 2.3**[27]. Let \( I^X \) denote the set of all fuzzy sets on \( X \) and \( A \subseteq X \). A pair \((f,A)\) is called a fuzzy soft set over \( X \), where \( f : X \rightarrow I^X \) is a function. That is, for each \( a \in A \), \( f(a) = f_a : X \rightarrow I \) is a fuzzy set on \( X \).

**Definition 2.4**[29]. For two fuzzy soft sets \((f,A)\) and \((g,B)\) over a common universe \( X \), we say that \((f,A)\) is a fuzzy soft subset of \((g,B)\) if

1. \( A \subseteq B \) and
2. for all \( a \in A \), \( f_a \leq g_a \); implies \( f_a \) is a fuzzy subset of \( g_a \).

We denote it by \((f,A)\preceq (g,B)\). \((f,A)\) is said to be a fuzzy soft super set of \((g,B)\), if \((g,B)\) is a fuzzy soft subset of \((f,A)\). We denote it by \((f,A)\succeq (g,B)\).

**Definition 2.5**[29]. Two fuzzy soft sets \((f,A)\) and \((g,B)\) over a common universe \( X \) are said to be fuzzy soft equal, if \((f,A)\) is a fuzzy soft subset of \((g,B)\) and \((g,B)\) is a fuzzy soft subset of \((f,A)\).

**Definition 2.6**[29]. The union of two fuzzy soft sets of \((f,A)\) and \((g,B)\) over the common universe \( X \) is the fuzzy soft set \((h,C)\), where \( C = A \cup B \) and for all \( c \in C \),

\[
    h_c = \begin{cases} 
    f_c, & \text{if } c \in A - B \\
    g_c, & \text{if } c \in B - A \\
    f_c \vee g_c, & \text{if } c \in A \cap B
    \end{cases}
\]

We write \((f,A)\dot{\cup}(g,B) = (h,C)\).

**Definition 2.7**[29]. The intersection \((h,C)\) of two fuzzy soft sets \((f,A)\) and \((g,B)\) over a common universe \( X \), denoted \((f,A)\cap (g,B)\), is defined as \( C = A \cap B \), and \( h_c = f_c \wedge g_c \), for all \( c \in C \).

**Definition 2.8**[29]. The relative complement of a fuzzy soft set \((f,A)\) is the fuzzy soft set \((f^c,A)\), which is denoted by \((f,A)^c\), where \( f^c : A \rightarrow F(U) \) is a fuzzy set-valued function. That is, for each \( a \in A \), \( f^c(a) \) is a fuzzy set in \( U \), whose membership function is \( f_c^c(x) = 1 - f_a(x) \), for all \( x \in U \). Here \( f_a^c \) is the membership function of \( f^c(a) \).
Definition 2.9[16]. The difference \((h, C)\) of two fuzzy soft sets \((f, A)\) and \((g, B)\) over \(X\), denoted by \((f, A)\setminus(g, B)\), is defined as \((f, A)\setminus(g, B) = (f, A)\setminus (g, B)^c\).

For our convenience, we will use the notation \(f_A\) for fuzzy soft set instead of \((f, A)\).

Definition 2.10[37]. Let \(\tau\) be the collection of fuzzy soft sets over \(X\), then \(\tau\) is said to be a fuzzy soft topology on \(X\) if

1. \(\emptyset, X\) belong to \(\tau\).
2. If \((f_A)_i \in \tau\), for all \(i \in I\), then \(\bigvee_{i \in I}(f_A)_i \in \tau\).
3. \(f_A, g_B \in \tau\) implies that \(f_A \bigwedge g_B \in \tau\).

The triplet \((X, \tau, A)\) is called a fuzzy soft topological space over \(X\). Every member of \(\tau\) is called fuzzy soft open set. A fuzzy soft set is called fuzzy soft closed if and only if its complement is fuzzy soft open.

Definition 2.11[38]. Let \((X, \tau, A)\) be a fuzzy soft topological space over \(X\) and \(f_A\) be a fuzzy soft set over \(X\). Then

1. fuzzy soft interior of fuzzy soft set \(f_A\) over \(X\) is denoted by \((f_A)^o\) and is defined as the union of all fuzzy soft open sets contained in \(f_A\). Thus \((f_A)^o\) is the largest fuzzy soft open set contained in \(f_A\).
2. fuzzy soft closure of \(f_A\), denoted by \(\overline{f_A}\) is the intersection of all fuzzy soft closed super sets of \(f_A\). Clearly \(\overline{f_A}\) is the smallest fuzzy soft closed set over \(X\) which contains \(f_A\).

Definition 2.12 [22]. Let \((X, \tau, A)\) be a fuzzy soft topological space over \(X\). A fuzzy soft set \(f_A\) is called fuzzy soft semi-open, if there exists a fuzzy soft open set \(g_A\) such that \(g_A \subseteq f_A \subseteq \overline{g_A}\). The class of all fuzzy soft semi-open sets in \(X\) is denoted by FSSO\((X)\). Note that every fuzzy soft open set is fuzzy soft semi-open but the converse is not true in general.

Definition 2.13[22]. A fuzzy soft set \(f_A\) in fuzzy soft topological space \((X, \tau, A)\) is fuzzy soft semi-closed if and only if its complement \((f_A)^c\) is fuzzy soft semi-open. The class of fuzzy soft semi-closed sets is denoted by FSSC\((X)\).

Note that every fuzzy soft closed set is fuzzy soft semi-closed in fuzzy soft topological space \((X, \tau, A)\).

Proposition 2.14[22]. Let \(f_A\) be a fuzzy soft set in fuzzy soft topological space \((X, \tau, A)\). Then \(f_A\) is fuzzy soft semi-closed if and only if there exists a fuzzy soft closed set \(h_A\) such that \((h_A)^c \subseteq f_A \subseteq h_A\).

Definition 2.15[22]. Let \(f_A\) be a fuzzy soft set in fuzzy soft topological space \((X, \tau, A)\). The fuzzy soft semi-closure of \(f_A\), denoted by \(scf^s(f_A)\) and is defined as the intersection of all fuzzy soft semi-closed supersets of \(f_A\). It is clear from the definition that \(scf^s(f_A)\) is the smallest fuzzy soft semi-closed set over \(X\) which contains \(f_A\).

Definition 2.16[22]. Let \(f_A\) be a fuzzy soft set in fuzzy soft topological space \((X, \tau, A)\). The fuzzy soft semi-interior of \(f_A\), denoted by \(sint^s(f_A)\) and is defined as the union of all fuzzy soft semi-open subsets of \(f_A\). It is clear from the definition that \(sint^s(f_A)\) is the largest fuzzy soft semi-open set over \(X\) contained in \(f_A\).

Definition 2.17[23]. Let \((X, \tau, A)\) be a fuzzy soft topological space over \(X\). Then a fuzzy soft set \(f_A\) over \(X\) is said to be a fuzzy soft pre-open, if \(f_A \subseteq \overline{(f_A)^c}\).

Definition 2.18[23]. Let \((X, \tau, A)\) be a fuzzy soft topological space over \(X\). Then a fuzzy soft set \(f_A\) over \(X\) is said to be a fuzzy soft pre-closed, if \((f_A)^c \subseteq \overline{(f_A)^c}\).
3. Fuzzy soft semi-pre-open(closed) sets

**Definition 3.1.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$, where $X$ is a nonempty set and $\tau$ is a family of fuzzy soft sets. Then a fuzzy soft set $f_A$ is said to be a fuzzy soft semi-pre-open, if there exists a fuzzy soft pre-open set $g_A$ such that $g_A \preceq f_A \preceq (g_A)$.  

**Definition 3.2.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$, where $X$ is a nonempty set and $\tau$ is a family of fuzzy soft sets. Then a fuzzy soft set $f_A$ is said to be a fuzzy soft semi-pre-closed, if there exists a fuzzy soft pre-closed set $g_A$ such that $(g_A) \preceq f_A \preceq g_A$. Note that the fuzzy soft set $f_A$ is fuzzy soft semi-pre-open if and only if $f_A^\circ$ is fuzzy soft semi-pre-closed.

**Remark 3.3.** It is clear that any fuzzy soft semi-open as well as fuzzy soft pre-open set is a fuzzy soft semi-pre-open set.

The following example shows that the converse of above remark is not true in general. For this we consider the Example 3.3[22] as:

**Example 3.4.** Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and $\tau = \{\bar{0}, \bar{1}, (f_A)_1, (f_A)_2, (f_A)_3, (f_A)_4\}$ where $(f_A)_1, (f_A)_2, (f_A)_3, (f_A)_4$ are fuzzy soft sets over $X$, defined as follows:

- $f_1(e_1)(h_1) = 0.5$, $f_1(e_1)(h_2) = 0.3$, $f_1(e_1)(h_3) = 0.2$,
- $f_1(e_2)(h_1) = 0.3$, $f_1(e_2)(h_2) = 0.5$, $f_1(e_2)(h_3) = 0.2$,
- $f_2(e_1)(h_1) = 1$, $f_2(e_1)(h_2) = 0$, $f_2(e_1)(h_3) = 0.5$,
- $f_2(e_2)(h_1) = 0.5$, $f_2(e_2)(h_2) = 0.3$, $f_2(e_2)(h_3) = 1$,
- $f_3(e_1)(h_1) = 0.5$, $f_3(e_1)(h_2) = 0$, $f_3(e_1)(h_3) = 0.2$,
- $f_3(e_2)(h_1) = 0.3$, $f_3(e_2)(h_2) = 0.3$, $f_3(e_2)(h_3) = 0.2$,
- $f_4(e_1)(h_1) = 1$, $f_4(e_1)(h_2) = 0.3$, $f_4(e_1)(h_3) = 0.5$,
- $f_4(e_2)(h_1) = 0.5$, $f_4(e_2)(h_2) = 0.5$, $f_4(e_2)(h_3) = 1$.

Then $\tau$ is a fuzzy soft topology on $X$ and hence $(X, \tau, A)$ is a fuzzy soft topological space over $X$.

Note that the fuzzy soft closed sets are

- $\{h_0.5, h_0.7, h_0.8\}, \{h_0.7, h_0.5, h_0.8\}$, $\{h_0.5, h_0.1, h_0.9\}, \{h_0.9, h_0.5, h_0.1\}$,
- $\{h_0.5, h_0.3, h_0.8\}, \{h_0.3, h_0.5, h_0.8\}$, $\{h_0.5, h_0.3, h_0.9\}, \{h_0.9, h_0.3, h_0.1\}$, $\bar{1}$ and $\bar{0}$.

Let us take fuzzy soft set $f_A$ over $X$ defined by

- $f(e_1)(h_1) = 0.3$, $f(e_1)(h_2) = 0.2$, $f(e_1)(h_3) = 0.1$,
- $f(e_2)(h_1) = 0.2$, $f(e_2)(h_2) = 0.3$, $f(e_2)(h_3) = 0.1$.

That is, $f_A = \{h_0.3, h_0.2, h_0.1\}$. Then there exists fuzzy soft set $g_A$ over $X$ defined by

- $g(e_1)(h_1) = 0.2$, $g(e_1)(h_2) = 0.1$, $g(e_1)(h_3) = 0.1$,
- $g(e_2)(h_1) = 0.2$, $g(e_2)(h_2) = 0.3$, $g(e_2)(h_3) = 0.1$.

That is, $g_A = \{h_0.2, h_0.1, h_0.9\}$. Since $g_A \preceq f_A \preceq g_A$, then $g_A$ is fuzzy soft pre-open set over $X$. Moreover, calculations show that

- $g_A \preceq f_A \preceq (g_A) \preceq \{h_0.5, h_0.3, h_0.2\}$, $\{h_0.3, h_0.5, h_0.2\}$, then $g_A$ is fuzzy soft pre-open set over $X$. Moreover, calculations show that

- $g_A \preceq f_A \preceq (g_A)$ implies that $f_A$ is fuzzy soft semi-pre-open set. But $f_A$ is not fuzzy soft semi-open set, since there does not exist any fuzzy soft open set $h_A$ such that $h_A \preceq f_A \preceq h_A$. Moreover, $f_A$ is not a fuzzy soft pre-open set.

**Lemma 3.5.** Let $(X, \tau, A)$ be fuzzy soft topological space over $X$ and $f_A$ be a fuzzy soft set over $X$. Then $f_A \preceq (f_A)^\circ$ if and only if $f_A \preceq (f_A)^\circ$.

**Proof.** $f_A \preceq (f_A)^\circ$ follows that $f_A \preceq (f_A)^\circ \preceq (f_A)^\circ$. Therefore, $f_A \preceq (f_A)^\circ$. The other inclusion follows similarly.

Using Proposition 3.4[22] and above Lemma 3.5, we have

**Theorem 3.6.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$ and $f_A$ be a fuzzy
soft set over $X$. Then the following statements are equivalent.
(1) $f_A$ is fuzzy soft semi-open.
(2) $f_A \leq (f_A)^\circ$.
(3) $F_A = (f_A)^\circ$.

The proof of the following theorem follows directly by taking complements in above Theorem 3.6.

**Theorem 3.7.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$ and $f_A$ be a fuzzy soft set over $X$. Then the following statements are equivalent.
(1) $f_A$ is fuzzy soft semi-closed.
(2) $f_A \leq (f_A)^\bar{\circ}$.
(3) $f_A \leq (f_A)^{\circ}$.

**Theorem 3.8.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$ and $f_A$ is fuzzy soft semi-pre-open set over $X$. Then the following hold.
(1) If $f_A$ is fuzzy soft semi-pre-open, then $f_A \leq (f_A)^{\circ}$.
(2) If $f_A$ is fuzzy soft semi-pre-closed, then $((f_A)^{\circ})^{\circ} \leq f_A$.

**Proof.** (1) $f_A$ is fuzzy soft semi-pre-open implies that there exists a fuzzy soft open set $g_A$ such that $g_A \leq f_A$. This follows that $f_A \leq (f_A)^{\circ}$. Also $g_A$ is fuzzy soft pre-open set implies that $f_A \leq (f_A)^{\circ}$. Therefore, $f_A \leq (f_A)^{\circ}$.
(2) This can be proved in the same way as (1).

The following theorem directly follows from Theorems 3.6, 3.7 and 3.8.

**Theorem 3.9.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$ and $f_A$ be a fuzzy soft set over $X$. If $f_A$ is fuzzy soft semi-pre-open(closed) and fuzzy soft open(closed), then $f_A$ is fuzzy soft semi-open(closed).

The following lemma is easy to proof.

**Lemma 3.10.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$ and $f_A$ be a family of fuzzy soft set over $X$. Then $\bigvee F_A \leq \bigvee F_A$. Moreover, for finite case, $\bigvee F_A = \bigvee F_A$ and $\bigvee (F_A)^\circ = \bigvee (F_A)^\circ$.

**Theorem 3.11.** Let $(X, \tau, A)$ be a fuzzy soft topological space over $X$. Then
(1) Arbitrary union of fuzzy soft semi-pre-open sets is a fuzzy soft semi-pre-open set.
(2) Arbitrary intersection of fuzzy soft semi-pre-closed sets is a fuzzy soft semi-pre-closed set.

**Proof.** Suppose that $f_{A_i}$ be a family of fuzzy soft semi-pre-open set in fuzzy soft topological space over $X$. Then for each $f_{A_i}$, there exists a fuzzy soft pre-open set $g_{A_i}$ such that $g_{A_i} \leq f_{A_i} \leq (g_{A_i})^{\circ}$. Using Lemma 3.10 and Theorem 3.10[23], we have $\bigvee g_{A_i} \leq \bigvee f_{A_i} \leq \bigvee (g_{A_i})^{\circ}$, where $g_{A_i}$ is fuzzy soft pre-open set.
(2) This follows directly by taking complements in (1).

**Theorem 3.12.** Let $f_A$ be a fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then
(1) $scl^{f_A}(f_A) = f_A \bigvee (f_A)^\circ$.
(2) $sint^{f_A}(f_A) = f_A \bigwedge (f_A)^\circ$.
(3) $sint^{f_A}(scl^{f_A}(f_A)) = scl^{f_A}((f_A)^\circ)$.
(4) $scl^{f_A}(sint^{f_A}(f_A)) = sint^{f_A}((f_A)^\circ)$.

**Proof.** (1) and (2) directly follows from the definitions of fuzzy soft interior(closure) and fuzzy soft semi-interior(closure).
(3) Using (2), we have, $sint^{f_A}(scl^{f_A}(f_A)) = scl^{f_A}((f_A)^\circ) \leq scl^{f_A}(f_A) \bigwedge (f_A)^\circ$.
(4) Using (3), we have, $scl^{f_A}(scl^{f_A}(f_A)) = scl^{f_A}((f_A)^\circ) \leq scl^{f_A}(f_A) \bigwedge (f_A)^\circ.$
Also, \( sint^{fs}(scd^{fs}(f_A))\) \(\subseteq\) \(scd^{fs}(f_A)\) \(\subseteq\) \(scd^{fs}(f_A)\). This follows (3).

(4) Using (1), we have, \( scd^{fs}(sint^{fs}(f_A))\) \(\subseteq\) \(sint^{fs}(f_A)\) \(\subseteq\) \(sint^{fs}(f_A)\) \(\subseteq\) \(sint^{fs}(f_A)\). Thus, this proves (4).

\[ \text{Theorem 3.13.} \] Let \( f_A \) and \( g_A \) are fuzzy soft sets in fuzzy soft topological space \( (X, \tau, A) \) over \( X \). If \( f_A \) is fuzzy soft semi-pre-open and \( f_A \) \(\subseteq\) \( g_A \), then \( g_A \) is a fuzzy soft semi-pre-open set.

\[ \text{Proof.} \] \( f_A \) is fuzzy soft semi-pre-open implies that there exists a fuzzy soft semi-pre-open set \( k_A \) in \( (X, \tau, A) \) such that \( k_A \) \(\subseteq\) \( f_A \). As \( f_A \) \(\subseteq\) \( g_A \), \( k_A \) \(\subseteq\) \( f_A \) \(\subseteq\) \( g_A \). Moreover, \( k_A \) \(\subseteq\) \( k_A \) implies that \( g_A \) \(\subseteq\) \( k_A \). Therefore, \( k_A \) \(\subseteq\) \( g_A \). Hence \( g_A \) is fuzzy soft semi-pre-open.

\[ \text{Definition 3.14[23].} \] A fuzzy soft set \( f_A \) is said to be a fuzzy soft point in fuzzy soft topological space \( (X, \tau, A) \) denoted by \( e(f_A) \), if for the element \( e \in A \), \( f(e) \neq 0 \) and \( f(e) = 0 \) for all \( e \in A \) \(\setminus\) \( e \).

\[ \text{Definition 3.15[23].} \] The fuzzy soft point \( e(f_A) \) is said to be in the fuzzy soft set \( g_A \), denoted by \( e(f_A) \) \(\subseteq\) \( g_A \), if for the element \( e \in A \), \( f(e) \leq g(e) \). Clearly, every fuzzy soft set \( g_A \) can be expressed as the union of all fuzzy soft points which belong to \( g_A \).

\[ \text{Definition 3.16[23].} \] The fuzzy soft point \( e(f_A) \) is called the complement of a fuzzy soft point \( e(f_A) \) if for all \( e \in A \) \(\setminus\) \( e \), \( f(e) = 0 \) and \( f(e) \neq 0 \) for every element \( e \in A \).

\[ \text{Example 3.17[23].} \] Let \( X = \{h_1, h_2, h_3\} \), \( A = \{e_1, e_2\} \) and consider the fuzzy soft set \( (f_A)_1 \) over \( X \) is defined as follows:

\[
\begin{align*}
f_1(e_1)(h_1) &= 0.5, \\
f_1(e_1)(h_2) &= 0.3, \\
f_1(e_1)(h_3) &= 0.2, \\
f_2(e_2)(h_1) &= 0.3, \\
f_2(e_2)(h_2) &= 0.5, \\
f_2(e_2)(h_3) &= 0.2,
\end{align*}
\]

That is \( (f_A)_1 = \{h_1, h_2, h_3\} \). Then \( e((f_A)_1) = \{e_1 = \{h_0, h_1, h_2\}\} \) is a fuzzy soft point. Moreover, the complement \( e((f_A)_1) \) is \( e((f_A)_1) = \{e_1 = \{h_0, h_0, h_0, h_0\}\} \).

\[ \text{Remark 3.18[23].} \] Note that, if the soft point \( e(f_A) \) is in the soft set \( g_A \), then it is not necessary that the complement \( e(f_A) \) is in the soft set \( g_A \).

The following example verify the above remark.

\[ \text{Example 3.19[23].} \] Let \( X = \{h_1, h_2, h_3\} \), \( A = \{e_1, e_2\} \). Then it is to be noted that the fuzzy soft point \( e(f_A) = \{e_1 = \{h_0, h_0, h_0\}\} \) is contained in the fuzzy soft set \( g_A = \{h_0, h_0, h_0, h_0\} \). Now we can see that the complement of fuzzy soft point \( e((f_A)_1) \) is \( e((f_A)_1) = \{e_1 = \{h_0, h_0, h_0, h_0\}\} \) is not contained in the complement of fuzzy soft set \( g_A \).

\[ \text{Theorem 3.20.} \] Let \( f_A \) be a fuzzy soft set in fuzzy soft topological space \( (X, \tau, A) \) over \( X \). Then the following statements are equivalent:

1. \( f_A \) is fuzzy soft semi-pre-open.
2. For every fuzzy soft point \( e(g_A) \) in \( f_A \), there exists a fuzzy soft semi-pre-open set \( h_A \) such that \( e(g_A) \subseteq h_A \subseteq f_A \).

\[ \text{Proof.} \] (1) \(\Rightarrow\) (2) Suppose that \( f_A \) is fuzzy soft semi-pre-open. Then for every fuzzy soft point \( e(g_A) \) in \( f_A \), take \( h_A = f_A \). We have (2).

(2) \(\Rightarrow\) (1) Consider \( f_A \subseteq g_A \subseteq f_A \subseteq f_A \). Hence the proof.

\[ \text{Theorem 3.21.} \] Let \( f_A \) and \( k_A \) are fuzzy soft sets in fuzzy soft topological space \( (X, \tau, A) \) over \( X \). If \( h_A \) is fuzzy soft semi-pre-closed with \( (h_A)^c \subseteq k_A \subseteq h_A \), then \( k_A \) is fuzzy soft semi-pre-closed.

\[ \text{Proof.} \] This directly follows from Theorem 3.13.

\[ \text{Definition 3.22.} \] Let \( k_A \) be fuzzy soft set in fuzzy soft topological space \( (X, \tau, A) \) over \( X \) and \( e(f_A) \) be a fuzzy soft point. If there exists a fuzzy soft semi-pre-open set \( g_A \) with
$e(f_A) \in g_A \subseteq k_A$, then $k_A$ is called fuzzy soft semi-pre-neighborhood (nb) of a fuzzy soft point $e(f_A)$.

The proof of the following theorem directly follows from the definitions of fuzzy soft point, fuzzy soft semi-open and Theorem 3.11.

**Theorem 3.23.** Let $e(f_A)$ be a fuzzy soft point and $g_A$ be fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then $g_A$ is fuzzy soft semi-open if and only if $g_A$ is fuzzy soft semi-pre-open of $e(f_A)$.

**Definition 3.24.** Let $f_A$ be a fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$.

1. fuzzy soft semi-pre-interior of fuzzy soft set $f_A$ denoted by $F^{\text{psint}}(f_A)$ and is defined as $F^{\text{psint}}(f_A) = \cup g_A : g_A$ is fuzzy soft semi-open and $g_A \subseteq f_A$.

2. fuzzy soft semi-pre-closure of fuzzy soft set $f_A$ denoted by $F^{\text{pscl}}(f_A)$ and is defined as $F^{\text{pscl}}(f_A) = \cap g_A : g_A$ is fuzzy soft semi-pre-closed and $f_A \subseteq g_A$.

**Theorem 3.25.** Let $f_A$ be a fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$.

1. $F^{\text{pscl}}(f_A) \subseteq (F^{\text{pscl}}(f_A))^c$.

2. $F^{\text{psint}}(f_A)^c \subseteq (F^{\text{pscl}}(f_A))^c$.

**Proof.** The proof follows from the Definition 3.24 and rule of complements.

**Theorem 3.26.** Let $e(f_A)$ be a fuzzy soft point and $g_A$ be fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then $e(f_A) \in F^{\text{pscl}}(g_A)$ if and only if $h_A \subseteq g_A$, for every fuzzy soft semi-pre-nb $h_A$.

**Proof.** $(\Rightarrow)$ Contrarily suppose that there exists fuzzy soft semi-nb $h_A$ of $e(f_A)$ such that $h_A \not\subseteq g_A$. Then there exists a fuzzy soft semi-open set $k_A$ such that $e(f_A) \in k_A \subseteq h_A$ and $k_A \not\subseteq g_A$. Now $k_A$ is fuzzy soft semi-pre-closed set with $g_A \subseteq k_A$ implies that $F^{\text{pscl}}(g_A) \subseteq k_A$. Also $e(f_A) \in k_A$ follows that $e(f_A) \not\in F^{\text{pscl}}(g_A)$. A contradiction.

$(\Leftarrow)$ Contrarily suppose that $e(f_A) \not\in F^{\text{pscl}}(g_A)$. Then there exists a fuzzy soft semi-pre-closed set $k_A$ such that $e(f_A) \not\in k_A$ and $g_A \subseteq k_A$. Therefore, $k_A$ is fuzzy soft semi-open set such that $e(f_A) \in k_A$ with $g_A \not\subseteq k_A$. A contradiction. This completes the proof.

**Theorem 3.27.** Let $f_A$ be a fuzzy soft set and $g_A$ be a fuzzy soft semi-pre-open set in fuzzy soft topological space $(X, \tau, A)$ over $X$. If $f_A \not\subseteq g_A$, then $F^{\text{pscl}}(f_A) \not\subseteq g_A$.

**Proof.** Contrarily suppose that $F^{\text{pscl}}(f_A) \subseteq g_A$. Then there exists a fuzzy soft point $e(h_A)$ in $(X, \tau, A)$ such that $F^{\text{pscl}}(f_A(e(h_A))) \cap (g_A(e(h_A))) \not\subseteq h_A$. Take $F^{\text{pscl}}(f_A(e(h_A))) \not\subseteq e(h_A)$. Then $g_A$ is a fuzzy soft semi-pre-nb of $e(h_A)$ with respect to $e(h_A)$ such that $f_A \not\subseteq g_A$. Therefore, $e(h_A) \not\in F^{\text{pscl}}(f_A)$. A contradiction. Hence the proof.

**Theorem 3.28.** Let $f_A$ be a fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then $F^{\text{pscl}}(f_A) = f_A \cap \overline{f_A}$.

**Proof.** Note that $\overline{f_A} = \overline{(f_A \cap (f_A)^c)} = \overline{f_A} \setminus (f_A \cap (f_A)^c)$.

Moreover, $F^{\text{pscl}}(f_A)$ is fuzzy soft semi-pre-closed implies that $(f_A \cap (f_A)^c) \subseteq (F^{\text{pscl}}(f_A))^c \subseteq F^{\text{pscl}}(f_A)$. Hence $f_A \cap \overline{f_A} \subseteq f_A \cap (f_A \cap (f_A)^c)^c = f_A \cap \overline{(f_A \cap (f_A)^c)}$. This completes the proof.

**Theorem 3.29.** Let $f_A$ be a fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then $F^{\text{psint}}(f_A) = f_A \cap \overline{f_A}$.

**Proof.** Note that $f_A \cap \overline{f_A} = (f_A \cap (f_A)^c)$.
Also, since $F(3)$
Proof. $F(3)$

The following theorem gives us the useful characterization of fuzzy soft semi-pre-open sets.

**Theorem 3.32.** Let $f_A$ be a fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then $F(pint^*(F^*pcl^*(f_A)) \leq F^*pcl^*(f_A)).$

**Proof.** Using Theorems 3.12(3)(4) and 3.30, we have

$$F(pint^*(F^*pcl^*(f_A)) \leq F^*pcl^*(f_A)) \leq (f_A \wedge ((F^*pcl^*(f_A))^{\circ})) \leq (f_A \wedge (\bigvee \{(f_A)^{\circ}\})) \leq F(pint^*(f_A)).$$

This completes the proof.

**Corollary 3.31.** Let $f_A$ be a fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then

1. $f_A \vee F^*pint^*(F^*pcl^*(f_A)) \leq F^*pcl^*(f_A)$.
2. $f_A \wedge F^*pint^*(F^*pcl^*(f_A)) \leq F^*pint^*(f_A)$.

**Theorem 3.33.** Let $f_A$ be a fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then the following statements are equivalent.

1. $f_A$ is fuzzy soft semi-pre-open.
2. $f_A \leq F^*pint^*(F^*pcl^*(f_A))$.
3. $f_A \leq sint^{f_A}(F^* pcl^*(f_A))$.

**Proof.** (1) $\Rightarrow$ (2) Suppose that $f_A$ is a fuzzy soft semi-pre-open set. Then $f_A \leq F^*pint^*(f_A) \leq F^*pint^*(F^*pcl^*(f_A))$.

(2) $\Rightarrow$ (3) This follows directly from Theorem 3.32.

(3) $\Rightarrow$ (1) Consider $f_A \leq sint^{f_A}(F^* pcl^*(f_A)) \leq F^*pcl^*(f_A)$. This follows that $f_A \leq (f_A)^{\circ}$. Hence $f_A$ is fuzzy soft semi-pre-open. This completes the proof.

The following theorem can be easily verified by using definitions.

**Theorem 3.34** Let $f_A$ and $g_A$ are fuzzy soft set in fuzzy soft topological space $(X, \tau, A)$ over $X$. Then

1. $F^*pcl^*(f_A) \leq f_A$ and $(f_A)^{\circ} \geq F^*pint^*(f_A)$.
2. $F^* pcl^*(f_A) \leq F^*pint^*(f_A)$ is fuzzy soft semi-pre-closed(open).
3. $f_A$ is fuzzy soft semi-pre-closed(open) if and only if $F^* pcl^*(f_A) \equiv f_A(F^*pint^*(f_A) \equiv f_A)$.
4. $f_A \leq g_A$ implies that $F^*pcl^*(f_A) \leq F^*pcl^*(g_A)$ and $F^*pint^*(f_A) \leq F^*pint^*(g_A)$.

4. **Fuzzy soft semi-pre-continuous functions**

In this section, we define and explore the characterizations of fuzzy soft semi-pre-continuous and fuzzy soft semi-pre-open(closed) mappings in a fuzzy soft topological spaces. First we recall some definitions.
**Definition 4.1[2].** Let $F(X, A)$ and $F(Y, B)$ be families of fuzzy soft sets. $u : X \to Y$ and $p : A \to B$ are mappings. Then image and inverse image of a function $f_{pu} : F(X, A) \to F(Y, B)$ is defined as:

1. Let $f_A$ be a fuzzy soft set in $F(X, A)$. The image of $f_A$ under $f_{pu}$, written as $f_{pu}(f_A)$, is a fuzzy soft set in $F(Y, B)$ such that for $\beta \in p(A) \subseteq B$ and $y \in Y$,

$$f_{pu}(f_A)(\beta) = \bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap A} (f_A(\alpha))(y),$$

for all $y \in B$. $f_{pu}(f_A)$ is known as a fuzzy soft image of a fuzzy soft set $f_A$.

2. Let $g_B$ be a fuzzy soft set in $F(Y, B)$. Then the fuzzy soft inverse image of $g_B$ under $f_{pu}$, written as $f_{pu}^{-1}(g_B)$, is a fuzzy soft set in $F(X, A)$ such that

$$f_{pu}^{-1}(g_B)(\alpha)(x) = \begin{cases} g_B(p(\alpha))(u(x)), & \text{if } p(\alpha) \in B \\ 0, & \text{otherwise} \end{cases},$$

for all $x \in A$. $f_{pu}^{-1}(g_B)$ is known as a fuzzy soft inverse image of a fuzzy soft set $g_B$.

The fuzzy soft function $f_{pu}$ is called fuzzy soft surjective, if $p$ and $u$ are surjective. The fuzzy soft function $f_{pu}$ is called fuzzy soft injective, if $p$ and $u$ are injective.

**Definition 4.2[23].** Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two fuzzy soft topological spaces.

1. A fuzzy soft mapping $f_{pu} : F(X, A) \to F(Y, B)$ is said to be fuzzy soft semi-continuous, if for any fuzzy soft open set $g_B$ in $(Y, \tau_2, B)$, $f_{pu}(g_B)$ is fuzzy soft semi-open in $(X, \tau_1, A)$.

2. A fuzzy soft mapping $f_{pu} : F(X, A) \to F(Y, B)$ is said to be a fuzzy soft semi-open, if for any fuzzy soft open set $f_A$ in $(X, \tau_1, A)$, $f_{pu}(f_A)$ is fuzzy soft semi-open in $(Y, \tau_2, B)$.

Now we define:

**Definition 4.3.** Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ is a fuzzy soft mapping. Then $f_{pu} : F(X, A) \to F(Y, B)$ is said to be fuzzy soft semi-pre-continuous, if for every fuzzy soft open set $g_B$ in $(Y, \tau_2, B)$, $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-pre-open in $(X, \tau_1, A)$.

**Remark 4.4.** Every fuzzy soft semi-continuous function is fuzzy soft semi-pre-continuous, since every fuzzy soft semi-open set is fuzzy soft semi-pre-open in fuzzy soft topological space. But the converse is not true in general.

The following theorem directly follows from Definition 4.3.

**Theorem 4.5.** Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ is a fuzzy soft mapping. Then the following statements are equivalent:

1. $f_{pu}$ is fuzzy soft semi-pre-continuous.
2. For every fuzzy soft closed set $g_B$ in $(Y, \tau_2, B)$, $f_{pu}^{-1}(g_B)$ is fuzzy soft semi-pre-closed in $(X, \tau_1, A)$.

**Theorem 4.6.** Let $(X, \tau_1, A)$ and $(Y, \tau_2, B)$ be two fuzzy soft topological spaces and $f_{pu} : F(X, A) \to F(Y, B)$ is a fuzzy soft mapping. Then the following statements are equivalent:

1. $f_{pu}$ is fuzzy soft semi-pre-continuous.
2. For every fuzzy soft point $e(f_A)$ in $(X, \tau_1, A)$ and every fuzzy soft open set $g_A$ with $f_{pu}(e(f_A)) \preceq g_A$, there is fuzzy soft semi-pre-open set $h_A$ with $e(f_A) \preceq h_A$ and $f_{pu}(h_A) \preceq g_A$.

**Proof.** (1) $\Rightarrow$ (2) Suppose that $e(f_A)$ be a fuzzy soft point in $(X, \tau_1, A)$ and $g_A$ be fuzzy
soft open in \((Y, \tau_2, B)\) such that \(f_{pu}(e(f_A)) \in g_A\). Take \(h_A \triangleq f_{pu}^{-1}(g_A)\). Then by (1), \(h_A\) is fuzzy soft semi-pre-open in \((X, \tau_1, A)\) with \(e(f_A) \in h_A\) and \(f_{pu}(h_A) \subseteq g_A\).

(2) \(\Rightarrow\) (1) Suppose that \(g_A\) be fuzzy soft open set in \((Y, \tau_2, B)\) and \(e(f_A) \in f_{pu}^{-1}(g_A)\). Then \(f_{pu}(e(f_A)) \subseteq g_A\). By (2), there is a fuzzy soft semi-pre-open set \(h_A\) in \((X, \tau_1, A)\) such that \(e(f_A) \in h_A\) and \(f_{pu}(h_A) \subseteq g_A\). This follows that \(e(f_A) \in h_A \triangleq f_{pu}^{-1}(g_A)\). Therefore, by Theorem 3.20, \(f_{pu}^{-1}(g_A)\) is fuzzy soft semi-pre-open. Hence \(f_{pu}\) is fuzzy soft semi-pre-continuous. This completes the proof.

**Definition 4.7**[21]. Let \(f_A\) be fuzzy soft set in fuzzy soft topological space \((X, \tau, A)\) over \(X\) and \(e(f_A)\) be fuzzy soft point. If there exists a fuzzy soft open set \(g_A\) with \(e(f_A) \in g_A\), then \(f_A\) is called fuzzy soft neighborhood(nbd) of a fuzzy soft point \(e(f_A)\).

**Theorem 4.8.** Let \((X, \tau_1, A)\) and \((Y, \tau_2, B)\) be two fuzzy soft topological spaces and \(f_{pu} : F(X, A) \to F(Y, B)\) is a fuzzy soft mapping. Then the following statements are equivalent:

1. \(f_{pu}\) is fuzzy soft semi-pre-continuous.
2. For every fuzzy soft point \(e(f_A)\) in \((X, \tau_1, A)\) and every fuzzy soft nbd \(g_A\) of \(f_{pu}(e(f_A))\), \(f_{pu}^{-1}(g_A)\) is fuzzy soft semi-pre-nbd of \(e(f_A)\).
3. For every fuzzy soft point \(e(f_A)\) in \((X, \tau_1, A)\) and every fuzzy soft nbd \(g_A\) of \(f_{pu}(e(f_A))\), there is fuzzy soft semi-pre-nbd \(k_A\) of \(e(f_A)\) such that \(f_{pu}(k_A) \subseteq g_A\).
4. For every fuzzy soft point \(e(f_A)\) in \((X, \tau_1, A)\) and every fuzzy soft open set \(g_A\) with \(f_{pu}(e(f_A)) \subseteq g_A\), there is fuzzy soft semi-pre-open set \(h_A\) with \(e(f_A) \in h_A\) and \(f_{pu}(h_A) \subseteq g_A\).

**Proof.** (1) \(\Rightarrow\) (2) Suppose that \(e(f_A)\) be a fuzzy soft point in \((X, \tau_1, A)\) and \(g_A\) be a fuzzy soft nbd of \(f_{pu}(e(f_A))\). This follows that there is a fuzzy soft open set \(k_A\) such that \(f_{pu}(e(f_A)) \subseteq k_A \subseteq g_A\). Now \(f_{pu}^{-1}(k_A)\) is fuzzy soft semi-pre-open and \(e(f_A) \subseteq f_{pu}^{-1}(k_A) \subseteq f_{pu}^{-1}(g_A)\). Therefore \(f_{pu}^{-1}(h_A)\) is fuzzy soft semi-pre-nbd of \(e(f_A)\) and \(f_{pu}(h_A) \subseteq f_{pu}(f_{pu}^{-1}(g_A)) \subseteq g_A\).

(2) \(\Rightarrow\) (3) Suppose that \(e(f_A)\) be a fuzzy soft point in \((X, \tau_1, A)\) and \(g_A\) be a fuzzy soft nbd of \(f_{pu}(e(f_A))\). This implies that \(k_A \subseteq f_{pu}^{-1}(g_A)\) is fuzzy soft semi-pre-nbd of \(e(f_A)\). (3) \(\Rightarrow\) (4) Let \(e(f_A)\) be a fuzzy soft point in \((X, \tau_1, A)\) and \(g_A\) be a fuzzy soft open set such that \(f_{pu}(e(f_A)) \subseteq g_A\). Then \(g_A\) is fuzzy soft nbd of \(f_{pu}(e(f_A))\). Therefore, there is fuzzy soft semi-pre-nbd \(k_A\) of \(e(f_A)\) in \((X, \tau_1, A)\) such that \(e(f_A) \in k_A\) and \(f_{pu}(k_A) \subseteq g_A\). Therefore, there is a fuzzy soft semi-pre-open set \(h_A\) such that \(e(f_A) \in h_A \subseteq k_A\). Hence \(f_{pu}(h_A) \subseteq f_{pu}(k_A) \subseteq g_A\).

(4) \(\Rightarrow\) (1) This follows from Theorem 4.6. Hence the proof.

**Theorem 4.9.** Let \((X, \tau_1, A)\) and \((Y, \tau_2, B)\) be two fuzzy soft topological spaces and \(f_{pu} : F(X, A) \to F(Y, B)\) is a fuzzy soft mapping. Then the following statements are equivalent:

1. \(f_{pu}\) is fuzzy soft semi-pre-continuous.
2. \(f_{pu}^{-1}((g_B)^c) \subseteq f^*\text{post}(f_{pu}^{-1}(g_B))\), for every fuzzy soft set \(g_B\) in \((Y, \tau_2, B)\).

**Proof.** (1) \(\Rightarrow\) (2) Suppose that \(g_B\) be fuzzy soft set in \((Y, \tau_2, B)\). Then \((g_B)^c\) is fuzzy soft open set. Since \(f_{pu} : F(X, A) \to F(Y, B)\) is fuzzy soft semi-pre-continuous, then \(f_{pu}^{-1}((g_B)^c)\) is fuzzy soft semi-pre-open in \((X, \tau_1, A)\). Therefore, \(f_{pu}^{-1}((g_B)^c) \subseteq f_{pu}^{-1}(g_B)\). Hence \(f_{pu}^{-1}((g_B)^c) \subseteq f^*\text{post}(f_{pu}^{-1}(g_B))\).

(2) \(\Rightarrow\) (1) This can be proved similarly. This completes the proof.

**Theorem 4.10.** Let \((X, \tau_1, A)\) and \((Y, \tau_2, B)\) be two fuzzy soft topological spaces and \(f_{pu} : F(X, A) \to F(Y, B)\) be a fuzzy soft mapping. Then the following statements are equivalent:

1. \(f_{pu}\) is fuzzy soft semi-pre-continuous.
2. \(f_{pu}(F^*\text{post}(f_A)) \subseteq (f_{pu}(f_A))^c\), for every fuzzy soft set \(f_A\) in \((X, \tau_1, A)\).
3. \(f^*\text{post}(f_{pu}^{-1}(g_B)) \subseteq f_{pu}^{-1}(g_B)\), for every fuzzy soft set \(g_B\) in \((Y, \tau_2, B)\).

**Proof.** (1) \(\Rightarrow\) (2) Let \(f_A\) be a fuzzy soft set in \((X, \tau_1, A)\). Then \((f_{pu}(f_A))^c\) is fuzzy soft
closed. Since \( f_{pu} \) is fuzzy semi-pre-continuous, then by Theorem 4.5, \( f_{pu}^{-1}(fpu(fA)) \) is fuzzy soft semi-pre-closed. Thus \( f_{pu}^{-1}(fpu(fA)) \) is fuzzy soft semi-pre-closed. Hence \( f_{pu}^{-1}(fpu(fA)) \) is fuzzy soft semi-pre-closed. Now \( f_{A} \) is fuzzy soft semi-pre-closed implies that \( F^{scl}(fpu(fA)) \) is fuzzy soft semi-pre-closed. Since \( f_{pu}^{-1}(fpu(fA)) \Rightarrow (3) \), we have \( f_{pu}(F^{scl}(fpu(fA))) \leq f_{pu}(g_{A}) \). Then \( g_{A} \) is fuzzy soft semi-pre-closed. Hence \( f_{pu}(g_{A}) \) is fuzzy soft semi-pre-closed. Thus the converse is not true in general.

**Theorem 4.13.** Let \((X, \tau_{1})\) and \((Y, \tau_{2})\) be two fuzzy soft topological spaces and \( f_{A} \) be fuzzy soft set in \((X, \tau_{1})\). Then a fuzzy soft mapping \( f_{pu} : F(X, \tau_{1}) \rightarrow F(Y, \tau_{2}) \) is fuzzy soft semi-pre-open if and only if \( f_{pu}(f_{pu}(fA)) \leq f_{pu}(g_{pu}(fpu(fA))) \). Our hypothesis follows that \( f_{pu}(fA) \) is fuzzy soft semi-pre-open in \((Y, \tau_{2}, B)\). Hence \( f_{pu}(fA) \) is fuzzy soft semi-pre-open in \((Y, \tau_{2}, B)\).

**Proof.** \((\Rightarrow)\) Suppose \( f_{pu} \) is fuzzy soft semi-pre-open and also for any fuzzy soft set \( f_{A} \) in \((X, \tau_{1}, A)\), we get \( f_{pu}(fpu(fA)) \leq f_{pu}(fpu(fA)) \). Our hypothesis follows that \( f_{pu}(fA) \) is fuzzy soft semi-pre-open in \((Y, \tau_{2}, B)\). Using Theorem 3.33, we have \( f_{pu}(fpu(fA)) \leq sint^{scl}(fpu(fpu(fA))) \).

\((\Leftarrow)\) Suppose that \( f_{pu}(fpu(fA)) \leq sint^{scl}(fpu(fpu(fA))) \), for fuzzy soft set \( f_{A} \) in \((X, \tau_{1}, A)\). Then \( g_{A} \) be any fuzzy soft semi-open set in \((X, \tau_{1}, A)\). Now take \( g_{pu}(fpu(fpu(fA))) \leq f_{pu}(fpu(fpu(fA))) \). This follows that \( f_{pu}(g_{A}) \leq sint^{scl}(fpu(fpu(fA))) \). Theorem 3.33 implies that \( f_{pu}(g_{A}) \) is fuzzy soft semi-pre-open in \((Y, \tau_{2}, B)\). Therefore, \( f_{pu}(fpu(fA)) \) is fuzzy soft semi-pre-open function. This completes the proof.

**Theorem 4.14.** Let \((X, \tau_{1}, A)\) and \((Y, \tau_{2}, B)\) be two fuzzy soft topological spaces. Then fuzzy soft mapping \( f_{pu} : F(X, \tau_{1}) \rightarrow F(Y, \tau_{2}) \) is fuzzy soft semi-pre-open if and only \( f_{pu}(f_{pu}(fA)) \leq F^{scl}(fpu(fpu(fA))) \), for every fuzzy soft set \( f_{A} \) in \((X, \tau_{1}, A)\).

**Proof.** \((\Rightarrow)\) Suppose that \( f_{pu} \) is fuzzy soft semi-pre-open and \( f_{pu} \) be fuzzy soft set in \((X, \tau_{1}, A)\). Then \( f_{pu}(fpu(fA)) \) is fuzzy soft semi-pre-open. Thus, \( f_{pu}(fpu(fpu(fA))) \leq F^{scl}(fpu(fpu(fA))) \). Hence \( f_{pu}(fpu(fpu(fA))) \leq F^{scl}(fpu(fpu(fA))) \).

\((\Leftarrow)\) Suppose that \( f_{pu}(fpu(fA)) \leq f_{pu}(fpu(fpu(fA))) \). Thus \( f_{pu}(fpu(fpu(fA))) \) is fuzzy soft semi-pre-open in \((Y, \tau_{2}, B)\). Hence the proof.

**Definition 4.15.** Let \((X, \tau_{1}, A)\) be two fuzzy soft topological spaces and \( f_{A} \) be fuzzy soft set in \((X, \tau_{1}, A)\). Then a fuzzy soft mapping \( f_{pu} : F(X, \tau_{1}) \rightarrow F(Y, \tau_{2}) \) is fuzzy soft semi-closed if and only \( F^{scl}(fpu(fpu(fA))) \leq f_{pu}(fpu(fA)) \).

**Remark 4.16.** Every fuzzy soft semi-closed function is fuzzy soft semi-pre-closed but the converse is not true in general, since every fuzzy soft semi-closed set is fuzzy soft semi-pre-closed. 

**Theorem 4.17.** Let \((X, \tau_{1}, A)\) and \((Y, \tau_{2}, B)\) be two fuzzy soft topological spaces and \( f_{A} \) be fuzzy soft set in \((X, \tau_{1}, A)\). Then \( f_{pu} : F(X, \tau_{1}) \rightarrow F(Y, \tau_{2}) \) is fuzzy soft semi-pre-closed if and only \( F^{scl}(fpu(fpu(fA))) \leq f_{pu}(fpu(fA)) \).

**Proof.** \((\Rightarrow)\) Let \( f_{pu} \) be fuzzy soft semi-pre-closed and \( f_{A} \) be fuzzy soft set in \((X, \tau_{1}, A)\).
Then $f_{pu}(fA)$ is fuzzy soft semi-pre-closed in $(Y, \tau_2, B)$. Therefore, $f_{pu}(fA) \subseteq f_{pu}(fA)$ follows that $F^*p_{cl^*}(f_{pu}(fA)) \subseteq f_{pu}(fA)$.

$(\Leftarrow)$ Suppose that $g_A$ be fuzzy soft closed in $(X, \tau_1, A)$. Then $f_{pu}(gA) \supseteq f_{pu}(gA) \supseteq F^*p_{cl^*}(f_{pu}(gA))$. Also by definition of fuzzy soft semi-pre-closure, $f_{pu}(gA) \subseteq f_{pu}(gA)$. This implies that $f_{pu}(gA) \subseteq f_{pu}(gA)$. Therefore, $f_{pu}$ is a fuzzy soft semi-pre-closed mapping. This completes the proof.

Conclusion: The separation properties for fuzzy soft sets seem to be of special similarity to the problem of pattern discrimination. The fuzzification of soft set theory is very important topic in recent days. Every day new methods are developing in the literature using fuzzy soft sets from an imprecise multiobserver data for problems of decision making. In this paper, We initiated and explored the properties of fuzzy soft semi-pre-open(closed) sets. We observed that any fuzzy soft semi-open as well as fuzzy soft pre-open set is a fuzzy soft semi-pre-open set. But the converse is not true. We introduced and investigated fuzzy soft semi-pre-interior and fuzzy soft semi-pre-closure in fuzzy soft topological spaces. Moreover, we defined and studied the characterization of fuzzy soft semi-pre-continuous and fuzzy soft semi-pre-open(closed) mappings in fuzzy soft topological spaces, which generalized fuzzy soft semi-continuous and fuzzy soft semi-open(closed) mappings. In future studies, we may develop more properties with applications of fuzzy soft sets in optimization problems as well as the problems of pattern discrimination.

References


