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A NEW SEASONAL FUZZY TIME SERIES METHOD BASED ON THE MULTIPLICATIVE NEURON MODEL AND SARIMA

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Abstract

When fuzzy time series include a seasonal component, conventional fuzzy time series models are not sufficient. For such fuzzy time series, lagged variables which are around the period of the time series should also be included in the model. Determining the lagged variables which will be in the forecasting model is a vital issue. Also, defining fuzzy relations is another important issue in the fuzzy time series approach. When the number of fuzzy lagged variables is large, using artificial neural networks to define fuzzy relations makes the operations easier and increases the forecasting accuracy. In this study, in order to deal with the problem of determining the lagged variables, and defining the fuzzy relations, a novel seasonal fuzzy time series approach based on SARIMA and the multiplicative neuron model is proposed. In the proposed method, the SARIMA method is exploited to choose the fuzzy lagged variables and multiplicative neuron model is employed to establish the fuzzy relations. To show the applicability of the proposed method, it is applied to the invoice sum accrued to health service providers. For comparison, the data is also analyzed with other fuzzy time series approaches in the literature. It is observed that the proposed method has the best forecasting accuracy with respect to other methods.

Keywords: Fuzzy time series, SARIMA, Multiplicative neuron model, Forecasting.

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1. Introduction

Fuzzy time series approaches have received more and more attention in recent years [2]. Fuzzy time series approaches have been successfully applied to data such as stock exchange, temperature and enrollment which include uncertainty. The fuzzy time series approach was first proposed by Song and Chissom [8, 9, 10]. The first fuzzy time series model did not have the ability to model a seasonal component. However, Song [7] was the first to suggest a seasonal fuzzy time series method. Also, Egrioglu *et al.* [3] proposed a seasonal fuzzy time series approach. In [3], the fuzzy lagged variables are determined by using SARIMA. Furthermore, in the method introduced in [3], the residuals obtained from SARIMA are also used in the modelling process. Therefore, this fuzzy time series approach in [3] is the first moving average type method in the literature. In order to avoid complicated and time consuming computations, feed forward neural networks can be used to define the fuzzy relations Aladag *et al.* [1]. Hence, Egrioglu *et al.* [3] also utilized feed forward neural networks in the determination of the fuzzy relations.

In this study, to overcome problems in determining the lagged variables, defining the fuzzy relations and to increase forecasting accuracy, a novel seasonal fuzzy time series method is proposed by improving the method suggested by Egrioglu *et al.* [3]. In the proposed method, to choose the fuzzy lagged variables, SARIMA is employed like in [3]. To determine the fuzzy relations, the multiplicative neuron model, which was proposed by Yadav *et al.* [11], is used in the proposed approach. In addition, the particle swarm optimization method is utilized in the training process of the multiplicative neuron model. In order to show the performance of the proposed approach, the invoice sum accrued to health service providers is analyzed with the method. The data is also forecasted by using other fuzzy time series methods. As a result of the application, it is seen that the proposed method produces the most accurate forecasts.

In the next section, basic definitions of fuzzy time series are given. In Sections 3 and 4, the modified particle swarm optimization technique, and the multiplicative neuron model are briefly presented, respectively. Section 5 introduces the proposed seasonal fuzzy time series approach. The implementation and results obtained are given in Section 6. Finally, the last section concludes the paper.

2. Fuzzy time series

The definition of fuzzy time series was firstly introduced by Song and Chissom [8, 9, 10]. In contrast to conventional time series methods, various theoretical assumptions do not need to be checked in the fuzzy time series approach. The most important advantage of the fuzzy time series approach is to be able to work with a very small set of data and not to require the linearity assumption. General definitions of fuzzy time series are given as follows:

Let U be the universe of discourse, where $U = \{u_1, u_2, \dots, u_b\}$. A fuzzy set A_i of U is defined as $A_i = f_{A_i}(u_1)/u_1 + f_{A_i}(u_2)/u_2 + \cdots + f_{A_i}(u_b)/u_b$, where f_{A_i} is the membership function of the fuzzy set A_i; $f_{A_i} : U \to [0, 1]$. u_a is a generic element of the fuzzy set A_i; $f_{A_i}(u_a)$ represents the degree of belongingness of u_a to A_i; $f_{A_i}(u_a) \in [0, 1]$ and $1 \le a \le b$.

2.1. Definition. The definition of a fuzzy time series.

Let $Y(t)$ $(t = \dots, 0, 1, 2, \dots)$, a subset of the real numbers, be the universe of discourse on which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is the collection $f_1(t), f_2(t), \dots$ then $F(t)$ is called a *fuzzy time series* defined on $Y(t)$.

2.2. Definition. The definition of fuzzy relationship.

Assume that $F(t)$ is caused by $F(t-1)$. Then the relationship can be expressed as

$$F(t) = F(t-1) \circ R(t, t-1),$$

where $R(t, t-1)$ is the fuzzy relationship between $F(t)$ and $F(t-1)$, and "$\circ$" represents the "max-min" composition of fuzzy sets. To sum up, let $F(t-1) = A_i$ and $F(t) = A_j$. Then the fuzzy logical relationship between $F(t)$ and $F(t-1)$ can be denoted by $A_i \o A_j$, where A_i refers to the left-hand side and A_j refers to the right-hand side of the fuzzy logical relationship. Furthermore, these fuzzy logical relationships can be grouped to establish different fuzzy relationships.

The definition of the first order seasonal fuzzy time series forecasting model can be given as follows, see Song [7]:

2.3. Definition. Let $F(t)$ be a fuzzy time series. Assuming there exists seasonality in $\{F(t)\}$, the first order seasonal fuzzy time series forecasting model is defined as

$$F(t-m) \to F(t), \tag{1}$$

where m denotes the period.

In Egrioglu *et al.* [3], a seasonal fuzzy time series forecasting model is defined by taking the MA structure into consideration. The definition of this model is given as follows:

2.4. Definition. Let F and G be two fuzzy time series. $F(t)$ is caused by $F(t-m_1), \ldots, F(t-m_{k-1}), F(t-m_k), G(t-n_1), G(t-n_2), \ldots, G(t-n_{l-1}), G(t-n_l)$, where m_i $(i = 1, 2, \ldots, k)$ and n_j $(j = 1, 2, \ldots, l)$ are integers $(1 \le m_1 \le m_k,\ 1 \le n_1 \le n_l)$.

Thus, the $(k, l)^{\text{th}}$ order partial bivariate fuzzy time series forecasting model is represented by

$$\begin{aligned} &F(t-m_1), \ldots, F(t-m_{k-1}), F(t-m_k), G(t-n_1), \\ &\qquad G(t-n_2), \ldots, G(t-n_{l-1}), G(t-n_l) \to F(t). \end{aligned} \tag{2}$$

3. Modified particle swarm optimization

Particle swarm optimization, which is a population based heuristic algorithm, was first proposed by Kennedy and Eberhart [4]. A distinguishing feature of this heuristic algorithm is that it simultaneously examines different points in different regions of the solution space to find the global optimum solution. Local optimum traps can be avoided because of this feature. In the proposed approach, a modified particle swarm optimization algorithm is exploited as a training algorithm for the multiplicative neuron model. This algorithm includes a time varying inertia weight like in Ma *et al.* [5]. In a similar way, this algorithm also has a time varying acceleration coefficient like in Shi and Eberhart [6]. The modified particle swarm optimization algorithm can be given as follows:

3.1. Algorithm. Modified particle swarm optimization

Step 1. *The position of the k^{th} $(k = 1, 2, \ldots, pn)$ particle is randomly determined and kept in a vector X_k given as follows:*

$$X_k = \{x_{k,1}, x_{k,2}, \ldots, x_{k,d}\},\ k = 1, 2, \ldots, pn,$$

where $x_{k,i}$ $(i = 1, 2, \ldots, d)$ represents the i^{th} position of the k^{th} particle. pn and d represent the number of particles in a swarm and the number of positions, respectively.

Step 2. *Velocities are randomly determined and stored in a vector V_k as given below.*

$$V_k = \{v_{k,1}, v_{k,2}, \ldots, v_{k,d}\},\ k = 1, 2, \ldots, pn.$$

Step 3. *Depending on the evaluation function, particles Pbest and Gbest shown in* (3) *and* (4)*, respectively, are determined.*

$$(3) \qquad Pbest_k = (p_{k,1}, p_{k,2}, \ldots, p_{k,d}), \; k = 1, 2, \ldots, pn,$$
$$(4) \qquad Gbest = (p_{g,1}, p_{g,2}, \ldots, p_{g,d}),$$

where $Pbest_k$ *is a vector storing the positions corresponding to the* k^{th} *particle's best individual performance, and Gbest represents the particle which has the best evaluation function value found so far.*

Step 4. *Let* c_1 *and* c_2 *represent the cognitive and social coefficients, respectively, and let* w *be the inertia parameter. Let* (c_{1i}, c_{1f}), (c_{2i}, c_{2f}), *and* (w_1, w_2) *be intervals which includes possible values for* c_1, c_2 *and* w*, respectively. At each iteration, these parameters are calculated by using the formulas* (5), (6) *and* (7).

$$(5) \qquad c_1 = (c_{1f} - c_{1i}) \frac{t}{\max t} + c_{1i},$$
$$(6) \qquad c_2 = (c_{2f} - c_{2i}) \frac{t}{\max t} + c_{2i}$$
$$(7) \qquad w = (w_2 - w_1) \frac{maxt - t}{maxt} + w_1,$$

where maxt and t represent he maximum iteration number and the current iteration number, respectively.

Step 5. *Values of the velocities and positions are updated by using the formulas* (8) *and* (9)*, respectively.*

$$(8) \qquad v_{i,d}^{t+1} = \left[w \times v_{i,d}^{t} + c_1 \times rand_1 \times (p_{i,d} - x_{i,d}) + c_2 \times rand_2 \times (p_{g,d} - x_{i,d}) \right]$$
$$(9) \qquad x_{i,d}^{t+1} = x_{i,d} + v_{i,d}^{t+1},$$

where $rand_1$ *and* $rand_2$ *are random values from the interval [0 1].*

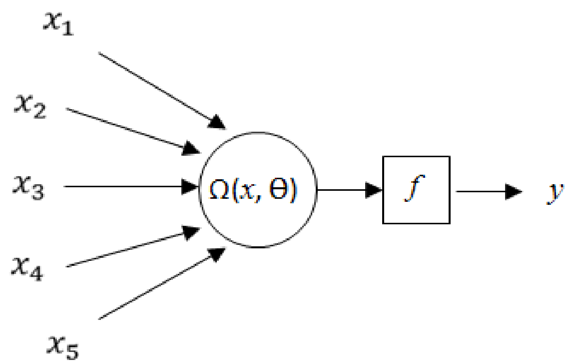
Step 6. *Steps 3 to 5 are repeated until a predetermined maximum iteration number (maxt) is reached.*

4. Multiplicative neuron model

The multiplicative neuron model is one type of artificial neural network. This neural network model was introduced in Yadav *et al.* [11] and it was shown that the model produces better forecasts. The multiplicative neuron model contains only one neuron. The structure of the model with five inputs is illustrated in Figure 1. In the model shown in Figure 1, the inputs are represented by x_i $(i = 1, \ldots, 5)$.

While inputs are summed in a feed forward neural network model, inputs are multiplied in a multiplicative neuron model. In other words, instead of using the sum function, the multiplication function is used in a multiplicative neuron model when input and output values are computed.

Figure 1. The multiplicative neuron model



The aggregation function consists of product of weighted inputs. f and y represent the activation function and the output of the model, respectively. In the literature, different learning algorithms have been utilized in the training process of a multiplicative neuron model. In this study, a particle swarm optimization method is used for training.

5. The proposed method

In the literature, there have been various methods which include first order, high order or multivariable models for forecasting fuzzy time series (Aladag *et al.* [2]). Since these models ignore seasonality, they can be insufficient for seasonal time series. A systematic approach was proposed by Egrioglu *et al.* [3] to forecast seasonal time series. The multiplicative neuron model can produce better results than a feed forward neural networks (Yadav *et al.* [11]). Therefore, in this study, the method suggested in Egrioglu *et al.* [3] is improved by using a multiplicative neuron model instead of a feed forward neural network. The proposed seasonal fuzzy time series forecasting approach is presented below.

5.1. Algorithm. The proposed approach.

Step 1. *By using the Box-Jenkins method, determine the SARIMA model.*

The residuals are calculated from this SARIMA model and then the inputs are determined.

For example, assume that the SARIMA(1,1,1)(0,1,1)$_{12}$ *model is found by analyzing a time series. Then, if* f *is considered as a linear function, the model can be symbolically written as follows:*

$$\begin{aligned} &SARIMA(1,1,1)(0,1,1)_{12}: \\ &X_t = f(X_{t-1}, X_{t-2}, X_{t-12}, X_{t-13}, X_{t-14}, a_{t-1}, a_{t-12}), \end{aligned} \tag{10}$$

where the residuals of time series are denoted by a_t.

Step 2. *Determine the order* (k, l) *of the model and the parameters* $m_1, \ldots, m_k$ *and* $n_1, \ldots, n_l$ *due to the inputs of the SARIMA model.*

For example, for the model SARIMA(1,1,1)(0,1,1)$_{12}$$(X_t = f(X_{t-1}, X_{t-2}, X_{t-12}, X_{t-13}, X_{t-14}, a_{t-1}, a_{t-12}))$ *the parameters, which also indicate the order, are* $k = 5$ *and*

$l = 2$. *A (5,2)*$^{\text{th}}$ *order partial bivariate fuzzy time series forecasting model can be given as follows:*

$$F(t-1), F(t-2), F(t-12), F(t-13), F(t-14),$$
$$G(t-1), G(t-12) \to F(t),$$

where $m_1 = 1, m_2 = 2, m_3 = 12, m_4 = 13, m_5 = 14, n_1 = 1,$ *and* $n_2 = 12$. $F(t)$ *and* $G(t)$ *denotes the fuzzy set* X_t *and fuzzy set* a_t*, respectively.*

Step 3. *Define the universe of discourses and subintervals for the time series and residuals.*

The min and max values of the time series are denoted by $D_{\min}$ *and* $D_{\max}$*, respectively. Then two positive numbers* D_1 *and* D_2 *can be chosen in order to define the universe of discourse* $U = [D_{\min} - D_1, D_{\max} + D_2]$.

To express this step more clearly, a small example is given as follows:

Assuming that $D_{\min} = 13055$ *and* $D_{\max} = 19377$*, the two positive numbers* $D_1 = 55$ *and* $D_2 = 663$ *can be chosen in order to divide the universe of discourse* U *evenly. Then,* $U = [13000, 20000]$ *is obtained. By choosing the length of the interval as 1000, subintervals* $u_1 = [13000, 14000]$, $u_2 = [14000, 15000]$, $u_3 = [15000, 16000]$, $u_4 = [16000, 17000]$, $u_5 = [17000, 18000]$, $u_6 = [18000, 19000]$, $u_7 = [19000, 20000]$ *are defined. In a similar manner, the universe of discourse* V *and subintervals* v_j *for the residuals are defined.*

Step 4. *Define fuzzy sets based on the universes of discourse.*

Based on the defined universe of discourse U *and the subintervals* V*, fuzzy sets* $A_1, A_2, \ldots, A_{k_1}$ *and* $B_1, B_2, \ldots, B_{k_2}$ *are defined as given below for the time series and residuals, respectively.*

$$A_1 = a_{11}/u_1 + a_{12}/u_2 + \cdots + a_{1n_1}/u_{r_1},$$
$$A_2 = a_{11}/u_1 + a_{12}/u_2 + \cdots + a_{1n_1}/u_{r_1},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$A_{k_1} = a_{11}/u_1 + a_{12}/u_2 + ... + a_{1n_1}/u_{r_1},$$
$$B_1 = b_{11}/v_1 + b_{12}/v_2 + \cdots + b_{1n_2}/v_{r_2},$$
$$B_2 = b_{11}/v_1 + b_{12}/v_2 + \cdots + b_{1n_2}/v_{r_2},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$B_{k_2} = b_{11}/v_1 + b_{12}/v_2 + \cdots + b_{1n_2}/v_{r_2},$$

where a_{ij} *is the degree of membership of* u_i *and* $a_{ij} \in [0,1]$, $1 \le i \le k_1$ *and* $1 \le j \le r_1$ *are defined in the given interval.* r_1 *and* k_1 *are the number of subintervals and of fuzzy sets, respectively. In a similar way, where* b_{ij} *gives the degrees of membership of* v_j, $b_{ij} \in [0,1]$, $1 \le i \le k_2$*, and* $1 \le j \le r_2$ *are defined in the given interval.* r_2 *and* k_2 *are the numbers of subintervals and of fuzzy sets, respectively.*

Step 5. *Fuzzify the actual data to fuzzy data.*

The residuals obtained from the SARIMA model and time series are fuzzified. The fuzzy time series is denoted by $F(t)$*, the fuzzy residuals from SARIMA by* $G(t)$.

Step 6. *Establish the fuzzy relationship.*

When establishing the fuzzy relationship the multiplicative neuron model can be employed. The data for training consists of the fuzzy time series lagged variables $F(t-m_1), \ldots, F(t-m_{k-1}), F(t-m_k)$ *and the fuzzy residuals lagged variables* $G(t-n_1), \ldots, G(t-n_{l-1}), G(t-n_l)$ *which are taken as the inputs of the network. The fuzzy time series* $F(t)$ *is used for the output of the network. The multiplicative neuron model is trained by modified particle swarm optimization in terms of these inputs and outputs.*

Step 7. *Calculate the forecast.*

Prepare the data for forecasting: $F(t+k-m_1),\ldots,F(t+k-m_{k-1}),F(t+k-m_k)$ *and* $G(t+k-n_1),\ldots,G(t+k-n_{l-1}),G(t+k-n_l)$ *are taken as the inputs for the trained multiplicative neuron model and the output from the model is the fuzzy forecast for* $F(t+k)$.

For example, for the model in (**??**)*, the inputs of the trained feed forward neural network are* $F(t+k-1),F(t+k-2),F(t+k-12),F(t+k-13),F(t+k-14),G(t+k-1)$ *and* $G(t+k-12)$*. Then, the forecast is* $F(t+k)$.

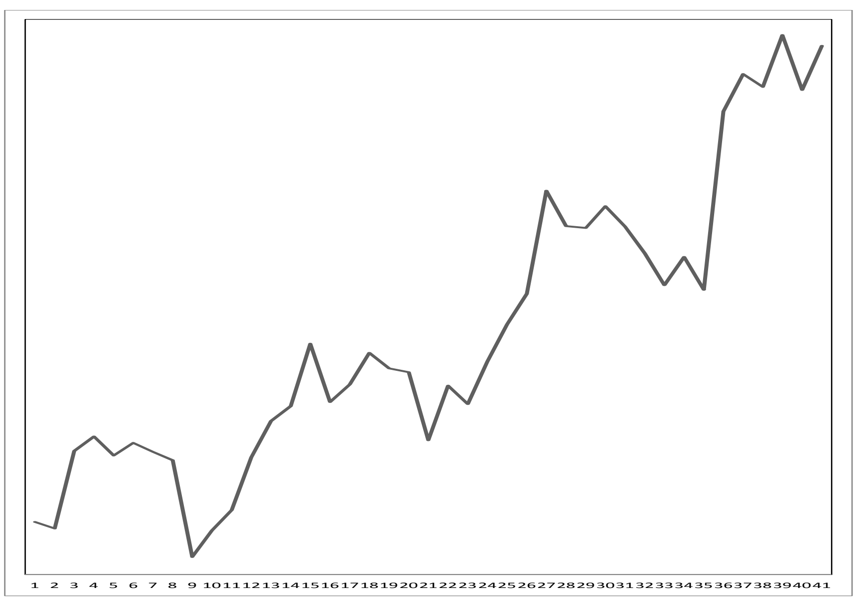
Step 8. *Defuzzify each fuzzy forecast* $F(t+k)$.

We applied the "Centroid" method to get the results. This procedure (also called center of area, center of gravity) is the most often adopted method of defuzzification. Suppose that the fuzzy forecast of $F(t+k)$ *is* A_k*. The defuzzified forecast is equal to the midpoint of the interval which corresponds to* A_k.

6. The implementation

The proposed seasonal fuzzy time series forecasting approach is applied to the invoice amount accrued to health service providers. Observations of this time series contains monthly observations are between January, 2008 and May, 2011. The graph of the time series can be seen in Fig. 2. The vertical and horizontal axes represent the invoice amount and observation number, respectively. The time series also analyzed with other fuzzy time series methods available in the literature for the aim of comparison. In all computations, Matlab version 2011 is utilized. Observations between January, 2008 and December, 2010 and between January, 2011 and May, 2011 are used for the training and test sets, respectively. In other words, the training and test sets consist of 36 and 5 observations, respectively.

Figure 2. The invoice sum accrued to health service providers



In the analysis, SARIMA and Winters additive seasonal exponential smoothing methods which are conventional time series approaches are used. Also, the method proposed by Egrioglu *et al.* [3], which is a fuzzy time series approach, is exploited. When SARIMA is used, it is found that SARIMA(0,1,0)(0,1,0) is the best model. Thus, $F(t-1), F(t-12)$, and $F(t-13)$ are taken as the fuzzy lagged variables in the proposed method. The forecasting model used in the proposed method can be obtained by taking $k=3$, $m_1=1$, $m_2=2$, $m_3=12$, and $l=0$ in (2).

In the fuzzification stage, the length of the interval is taken as 80 for both the method proposed in Egrioglu *et al.* [3] and that proposed in this study. In the phase of defining the fuzzy relations, the optimum weights of the multiplicative neuron model obtained by using modified particle swarm optimization are presented in Table 1. When feed forward neural networks are employed to determine the fuzzy relations in the method proposed by Egrioglu *et al.* [3], the inputs of the network are taken as $F(t-1)$, $F(t-12)$, and $F(t-13)$ due to the SARIMA(0,1,0)(0,1,0) model, and the number of neurons in the hidden layer is picked as 3.

All forecasts obtained from the methods and corresponding root mean square error values (RMSE) are presented in Table 2. RMSE can be calculated by using the formula given below.

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{n} (\text{forecast}_t - \text{actual}_t)^2}{n}},$$

where t represents the time and n is the number of observations in the test sets. forecast_t is the forecast at t from any mentioned model and actual_t is the actual value at t.

Table 1. Optimal weights of the multiplicative neuron model

W	b
2.56026808	-0.19308419
2.32612553	-0.60713836
-2.52094883	3.52470690

Table 2. The Results Obtained for the Test Set

Test Data	SARIMA	Winters Additive Model	Egrioglu *et al.* [3]	Proposed Method
1811,26	1821,44	1691,87	1708,65	1708,65
1790,43	1881,68	1701,79	1868,65	1788,65
1874,52	2059,22	1861,73	1868,65	1788,65
1785,58	2012,33	1837,67	1868,65	1788,65
1857,00	2019,85	1844,62	1868,65	1788,65
RMSE	155,23	70,90	68,87	67,21

7. Conclusion and discussion

The method proposed by Egrioglu *et al.* [3] is an effective method to forecast seasonal fuzzy time series. It was the first model that takes a moving average structure into consideration. In the model suggested in Egrioglu *et al.* [3], feed forward neural networks are exploited in the determination of the fuzzy relations.

By improving the method proposed by Egrioglu *et al.* [3], a new seasonal fuzzy time series forecasting method is proposed in this study in order to reach a high forecasting accuracy level. In the proposed approach, a multiplicative neuron model is utilized to define the fuzzy relations instead of a feed forward neural network. In addition, to train the multiplicative neuron model, a modified particle swarm optimization is employed.

To evaluate the forecasting performance of the proposed approach, the invoice sum accrued to health service providers is forecasted with both the proposed method and the other methods available in the literature. As a result of the implementation, it is seen that the most accurate forecasts are obtained when the proposed approach is used.

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