Improved ratio-type estimators of finite population variance using quartiles

Ramkrishna S. Solanki∗ † Housila P. Singh‡ and Surya K. Pal§

Abstract

In this paper we have proposed some ratio-type estimators of finite population variance using known values of parameters related to an auxiliary variable such as quartiles with their properties in simple random sampling. The suggested estimators have been compared with the usual unbiased and ratio estimators and the estimators due to [2], [12, 13, 14] and [3]. An empirical study is also carried out to judge the merits of the proposed estimator over other existing estimators of population variance using natural data set.

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1. Introduction

Estimating the finite population variance has great significance in various fields such as industry, agriculture, medical and biological sciences where we come across the populations which are likely to be skewed. Variation is present everywhere in our day to day life. It is law of nature that no two things or individuals are exactly alike. For instance, a physician needs a full understanding of variation in the degree of human blood pressure, body temperature and pulse rate for adequate prescription. A manufacture needs constant knowledge of the level of variation in people’s reaction to his product to be able to known whether to reduce or increase his price, or improve the quality of his product. An agriculturist needs an adequate understanding of variations in climate factors especially from place to place (or time to time) to be able to plan on when, how and where

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to plant his crop. In manufacturing industries and pharmaceutical laboratories some of
times researchers are more interested about the variation of their products or yields.
Many more situations can be encountered in practice where the estimation of population
variance of the study variable assumes importance. For these reasons various authors
have paid their attention towards the estimation of population variance. In sample sur-
veys, auxiliary information on the finite population under study is quite often available
from previous experience, census or administrative databases. The sampling literature
describes a wide variety of techniques for using auxiliary information to improve the
sampling design and/or obtain more efficient estimators of finite population variance. It
is well known that when the auxiliary information is to be used at the estimation stage,
the ratio method of estimation is extensively employed. The ratio estimation method
has been extensively used because of its intuitive appeal, computational simplicity and
applicability to a general design. Perhaps, this is why many researchers have directed
their efforts toward to get more efficient ratio-type estimators of the population variance
by modifying the structure of existing estimators. Such as, [2], [5], [6, 7], [8] and [11]
have suggested some modified estimators of population variance using known values of
coefficient of variation, coefficient of kurtosis, coefficient of skewness of an auxiliary vari-
able together with their biases and mean squared errors. We have known that the value
of quartiles and their functions are unaffected by the extreme values or the presence of
outliers in the population values. For this reason, [3] and [12, 13, 14] have considered the
problem of estimating the population variance of the study variable using information
on variance, quartiles, inter-quartile range, semi-quartile range and semi-quartile average
of an auxiliary variable. In this paper our main goal is to estimate the unknown popu-
lation variance of the study variable by improving the estimators suggested previously
using same information on an auxiliary variable such as quartiles, inter-quartile range,
semi-quartile range, semi-quartile average etc. The remaining part of the paper is orga-
nized as follows: The Section 2 introduced the notations and some existing estimators of
population variance in brief. In Section 3, the ratio-type estimator of population vari-
ance is suggested and the expressions of their asymptotic biases and the mean squared
errors are obtained. In addition, some members of suggested ratio-type estimators are
also generated with their properties. The Section 4 is addressed the problem of efficiency
comparisons of proposed ratio-type estimators with the usual unbiased estimator and the
estimator due to [1], while Section 5 is focused on empirical study of proposed ratio-type
estimators for the real data set. We conclude with a brief discussion in Section 6.

2. Notations and literature review

Much literature has been produced on sampling from finite populations to address
the issue of the efficient estimation of the variance of a survey variable when auxil-
iary variables are available. Our analysis refers to simple random sampling without
replacement (SRSWOR) and considers, for brevity, the case when only a single auxil-
iary variable is used. Let $U = (U_1, U_2, ..., U_N)$ be finite population of size $N$ and $(y, x)$
are (study, auxiliary) variables taking values $(y_i, x_i)$ respectively for the $i^{th}$ unit $U_i$ of
the finite population $U$. Our quest is to estimate the unknown population variance
$S_y^2 = (N - 1)^{-1} \sum_{i=1}^{N} (y_i - \bar{Y})^2$ of study variable $y$, where $\bar{Y} = N^{-1} \sum_{i=1}^{N} y_i$ is the
population mean of $y$. Let a simple random sample (SRS) of size $n$ be drawn without
replacement (WOR) from the finite population $U$. The usual unbiased estimator of finite
population variance $S_y^2$ is defined as

\begin{equation}
S_y^2 = t_0 = (n - 1)^{-1} \sum_{i=1}^{n} (y_i - \bar{y})^2
\end{equation}
where \( \bar{y} = n^{-1} \sum_{i=1}^{n} y_i \). [1] has suggested the usual ratio estimator of \( S_y^2 \) as

\[
(2.2) \quad t_R = t_1 = s_y^2 \left( \frac{S_y^2}{S_x^2} \right)
\]

where \( S_y^2 = (N-1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})^2 \), \( \bar{X} = N^{-1} \sum_{i=1}^{N} x_i \), \( s_y^2 = (n-1)^{-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \) and \( \bar{x} = n^{-1} \sum_{i=1}^{n} x_i \). Motivated by [10], [15] and [9], [2] have proposed following ratio-type estimators of the population variance as

\[
(2.3) \quad t_2 = s_y^2 \left( \frac{S_y^2 - C_x}{s_x^2 - C_x} \right)
\]

\[
(2.4) \quad t_3 = s_y^2 \left( \frac{S_y^2 - \beta_2(x)}{s_x^2 - \beta_2(x)} \right)
\]

\[
(2.5) \quad t_4 = s_y^2 \left( \frac{\beta_2(x) S_x^2 - C_x}{\beta_2(x) s_x^2 - C_x} \right)
\]

\[
(2.6) \quad t_5 = s_y^2 \left( \frac{C_x S_x^2 - \beta_2(x)}{C_x s_x^2 - \beta_2(x)} \right)
\]

where \( C_x = (S_x/\bar{X}) \) and \( \beta_2(x) \) are the known coefficients of variation and kurtosis of the auxiliary variable \( x \) respectively. Using the known value of population median \( Q_2 \) of the auxiliary variable \( x \) [12] have suggested the ratio-type estimator of population variance \( S_y^2 \) as

\[
(2.7) \quad t_6 = s_y^2 \left( \frac{S_y^2 + Q_2}{s_x^2 + Q_2} \right)
\]

[13] have proposed the modified ratio-type estimators of population variance \( S_y^2 \) of the study variable \( y \) using the known quartiles and their functions of the auxiliary variable \( x \) as

\[
(2.8) \quad t_7 = s_y^2 \left( \frac{S_y^2 + Q_1}{s_x^2 + Q_1} \right)
\]

\[
(2.9) \quad t_8 = s_y^2 \left( \frac{S_y^2 + Q_3}{s_x^2 + Q_3} \right)
\]

\[
(2.10) \quad t_9 = s_y^2 \left( \frac{S_y^2 + Q_c}{s_x^2 + Q_c} \right)
\]

\[
(2.11) \quad t_{10} = s_y^2 \left( \frac{S_y^2 + Q_d}{s_x^2 + Q_d} \right)
\]

\[
(2.12) \quad t_{11} = s_y^2 \left( \frac{S_y^2 + Q_a}{s_x^2 + Q_a} \right)
\]

where \( Q_i \) is the \( i^{th} \) quartile \( (i = 1, 3, Q_r = (Q_3 - Q_1) \) (inter-quartile range) \( Q_d = (Q_3 - Q_1) \) (semi-quartile range) and \( Q_a = (Q_3 + Q_1) \) (semi-quartile average). Taking motivation form [2] and [12]; [14] have suggested the ratio-type estimators of population variance \( S_y^2 \) using known values of coefficient of variation \( C_x \) and population median \( Q_2 \) of an auxiliary variable \( x \) as

\[
(2.13) \quad t_{12} = s_y^2 \left( \frac{C_x S_x^2 + Q_2}{C_x s_x^2 + Q_2} \right)
\]

Recently [3] have proposed another ratio-type estimator of population variance \( S_y^2 \) using known values of coefficient of correlation \( \rho \) between the variables \( (y, x) \) and population quartile \( Q_3 \) of an auxiliary variable \( x \) as

\[
(2.14) \quad t_{13} = s_y^2 \left( \frac{\rho S_y^2 + Q_3}{\rho s_x^2 + Q_3} \right)
\]
To the first degree of approximation the biases and mean squared errors (MSEs) of the estimators $t_j, (j = 0, 1, 2, ..., 13)$ are respectively given as

(2.15) \[ \text{Bias} (t_j) = \Phi \tau_j (\tau_j - c) \]

(2.16) \[ \text{MSE} (t_j) = \gamma [\lambda_{00} + \tau_j \lambda_{04} (\tau_j - 2c)] \]

where $\Phi = \gamma \lambda_{04}$, $\gamma = n^{-1} S^2_y$, $c$ = $\left( \lambda^2_{22} \lambda_{04}^{* - 1} \right)$, $\tau_0 = 0$, $\tau_1 = 1$, $\tau_2 = S^2_y (S^2_x - C_x)^{-1}$, $\tau_3 = S^2_y (S^2_x - \beta_2 (x))^{-1}$, $\tau_4 = \beta_2 (x) S^2_y (\beta_2 (x) S^2_x - C_x)^{-1}$, $\tau_5 = C_x S^2_y (C_x S^2_x - \beta_2 (x))^{-1}$, $\tau_6 = S^2_y (S^2_x + Q_1)^{-1}$, $\tau_7 = S^2_y (S^2_x + Q_4)^{-1}$, $\tau_8 = S^2_y (S^2_x + Q_1)^{-1}$, $\tau_9 = S^2_y (S^2_x + Q_4)^{-1}$, $\tau_{10} = S^2_y (S^2_x + Q_4)^{-1}$, $\tau_{11} = S^2_y (S^2_x + Q_a)^{-1}$, $\tau_{12} = C_x S^2_y (C_x S^2_x + Q_a)^{-1}$, $\tau_{13} = \rho S^2_y (\rho S^2_x + Q_4)^{-1}$, $\lambda^r_s = (\lambda_{rs} - 1)$, $\lambda_{rs} = \mu_{rs} \left( \mu_{02}^{\mu_{00}} \mu_{20}^{2} \right)^{-1}$, $\mu_{rs} = N^{-1} \sum_{i=1}^{N} (y_i - Y)^s (x_i - X)^r$ ($r, s$ being non negative integers). It is observed that the estimators $(t_0, t_7, ..., t_{13})$ due to [12, 13, 14] and [3] have used the quartiles and their functions such as inter-quartile range $Q_r$, semi-quartile range $Q_d$ and semi-quartile average $Q_a$ and in additive form to sample and population variances $S^2_x$ and $S^2_y$ respectively of the auxiliary variable $x$.

It is to be noted that the unit of the quartiles and their function as given above is of the study variable $y$. These lead authors to develop a more justified ratio-type estimators of the population variance $S^2_y$ of the study variable $y$ using known values of parameters related to the auxiliary variable $x$ and study their properties in simple random sampling.

3. The proposed ratio-type estimator

We propose following ratio-type estimators of population variance $S^2_y$ in simple random sampling as

(3.1) \[ T = S^2_y \left( \frac{\delta S^2_y + \alpha L^2}{\delta S^2_y + \alpha L^2} \right) \]

where \((\delta S^2_y + \alpha L^2) > 0, (\delta S^2_y + \alpha L^2) > 0 \text{ and } (\delta, L) \text{ are either real constants or function of known parameters of an auxiliary variable } x \text{ with } 0 \leq \alpha \leq 1 \). To obtain the bias and MSE of the proposed ratio-type estimator $T$, we write $S^2_y = S^2_y (1 + e_0)$ and $S^2_x = S^2_x (1 + e_1)$ such that $E (e_0) = E (e_1) = 0$ and to the first degree of approximation (ignoring finite population correction (f.p.c) term), we have $E (e_0^2) = n^{-1} \lambda_{00}$, $E (e_1^2) = n^{-1} \lambda_{04}$, $E (e_0 e_1) = n^{-1} \lambda_{04}$. Now expressing (3.1) in terms of $e$’s, we have

(3.2) \[ T = S^2_y (1 + e_0) \left( \frac{\delta S^2_y + \alpha L^2}{\delta S^2_y (1 + e_1) + \alpha L^2} \right) = S^2_y (1 + e_0) (1 + \tau e_1)^{-1} \]

where $\tau = \delta S^2_y (\delta S^2_y + \alpha L^2)^{-1}$. We assume that $|\tau e_1| < 1$ so that $(1 + \tau e_1)^{-1}$ is expandable. Expanding the right hand side of (3.2) and multiplying out, we have

\[
T = S^2_y (1 + e_0) \left( 1 - \tau e_1 + \tau^2 e_1^2 - \ldots \right) = S^2_y (1 + e_0 - \tau e_1 - \tau^2 e_0 e_1 + \tau^2 e_1^2 - \ldots)
\]

Neglecting terms of $e$’s having power greater than the two, we have $T \geq S^2_y (1 + e_0 - \tau e_1 - \tau^2 e_0 e_1 + \tau^2 e_1^2)$ or

(3.3) \[ (T - S^2_y) \geq S^2_y (e_0 - \tau e_1 - \tau^2 e_0 e_1 + \tau^2 e_1^2) \]

Taking expectation of both sides of (3.3), we get the bias of the estimator $T$ to the first degree of approximation as

(3.4) \[ \text{Bias} (T) = \Phi \tau (\tau - c) \]
If we set $T$ given by (3.13), we have

(3.5) $$(T - S_y^2)^2 \approx S_y^4 (c_0^2 + \tau^2 c_1^2 - 2\tau c_0 c_1)$$

Taking expectation of both sides of (3.5), we get the $MSE$ of the estimator $T$ to the first degree of approximation as

(3.6) $MSE(T) = \gamma [\lambda_{40} + \tau^2 \lambda_{04} (\tau^2 - 2c)]$

Below we have identified some members of proposed ratio type estimator $T$ for different choices of $(\delta, L)$.

(i) The estimator based on coefficient of variation $C_x$ and quartile $Q_1$:
If we set $(\delta, L) = (C_x, Q_1)$ in (3.1), we get the estimator of $S_y^2$ as,

(3.7) $T_1 = s_y^2 \left( \frac{C_x S_y^2 + \alpha Q_y^2}{C_x s_y^2 + \alpha Q_y^2} \right)$

(ii) The estimator based on coefficient of kurtosis $\beta_2(x)$ and median $Q_2$:
If we set $(\delta, L) = (\beta_2(x), Q_2)$ in (3.1), we get the estimator of $S_y^2$ as,

(3.8) $T_2 = s_y^2 \left( \frac{\beta_2(x) S_y^2 + \alpha Q_y^2}{\beta_2(x) s_y^2 + \alpha Q_y^2} \right)$

(iii) The estimator based on population mean $\bar{X}$ and quartile $Q_3$:
If we set $(\delta, L) = (\bar{X}, Q_3)$ in (3.1), we get the estimator of $S_y^2$ as,

(3.9) $T_3 = s_y^2 \left( \frac{\bar{X} S_y^2 + \alpha Q_y^2}{\bar{X} s_y^2 + \alpha Q_y^2} \right)$

(iv) The estimator based on coefficient of kurtosis $\beta_2(x)$ and inter-quartile range $Q_4$:
If we set $(\delta, L) = (\beta_2(x), Q_4)$ in (3.1), we get the estimator of $S_y^2$ as,

(3.10) $T_4 = s_y^2 \left( \frac{\beta_2(x) S_y^2 + \alpha Q_y^2}{\beta_2(x) s_y^2 + \alpha Q_y^2} \right)$

(v) The estimator based on correlation coefficient $\rho$ and semi-quartile range $Q_3$:
If we set $(\delta, L) = (\rho, Q_3)$ in (3.1), we get the estimator of $S_y^2$ as,

(3.11) $T_5 = s_y^2 \left( \frac{\rho S_y^2 + \alpha Q_y^2}{\rho s_y^2 + \alpha Q_y^2} \right)$

(vi) The estimator based on correlation coefficient $\rho$ and semi-quartile average $Q_4$:
If we set $(\delta, L) = (\rho, Q_4)$ in (3.1), we get the estimator of $S_y^2$ as,

(3.12) $T_6 = s_y^2 \left( \frac{\rho S_y^2 + \alpha Q_y^2}{\rho s_y^2 + \alpha Q_y^2} \right)$

Similarly one can identify many other estimators from the proposed ratio-type estimator $T$ for different combinations of $(\delta, L)$. To the first degree of approximation the biases and mean squared errors ($MSEs$) of the estimators $T_k, (k = 1, 2, ..., 6)$ are respectively given by

(3.13) $Bias(T_k) = \Phi \tau_k^* (\tau_k^* - c)$

(3.14) $MSE(T_k) = \gamma [\lambda_{40} + \tau_k^* \lambda_{04} (\tau_k^* - 2c)]$

where $\tau_1^* = C_x S_x^2 (C_x S_x^2 + \alpha Q_y^2)^{-1}, \tau_2^* = \beta_2(x) S_y^2 (\beta_2(x) S_y^2 + \alpha Q_y^2)^{-1}, \tau_3^* = \bar{X} S_y^2 (\bar{X} S_y^2 + \alpha Q_y^2)^{-1}, \tau_4^* = \beta_2(x) S_y^2 (\beta_2(x) S_y^2 + \alpha Q_y^2)^{-1}, \tau_5^* = \rho S_y^2 (\rho S_y^2 + \alpha Q_y^2)^{-1}, \tau_6^* = \rho S_y^2 (\rho S_y^2 + \alpha Q_y^2)^{-1}$. 
unbiased estimator \( \text{PRE} \) respectively as computed the percent relative efficiencies respectively for the population data set [Source: [4]] summarized in Table 1. We have the suggested ratio-type estimator and the estimators are summarized in Tables 2 and 3. It is observed from Tables 2 and 3 that all the ratio-type estimators \( T \) and the suggested ratio-type estimators \( T \) which are members of proposed ratio-type estimator are more efficient than the other existing estimators \( T \), \( j = 0, 1, \ldots, 13 \) which are due to [1], [2], [12, 13, 14] and [3] respectively. From (2.16) and (3.14), we have \( \text{MSE} (T_k) < \text{MSE} (T_j) \) if \( \tau_k^* (\tau_k^* - 2c) < \gamma_j (\gamma_j - 2c) \) i.e. if either, \( \tau_k^* < \gamma_j \) and \( c < \left( \frac{\gamma_j + \gamma_j}{2} \right) \) or, \( \tau_k^* > \gamma_j \) and \( c > \left( \frac{\gamma_j + \gamma_j}{2} \right) \) or equivalently , \( \min [\gamma_j, (2c - \gamma_j)] \leq \tau_k^* \leq \max [\gamma_j, (2c - \gamma_j)] , \) \( j = 0, 1, \ldots, 13; k = 1, 2, \ldots, 6 \).

4. The theoretical evaluation

We have made some theoretical conditions under which the ratio-type estimators \( T_k, (k = 1, 2, \ldots, 6) \) which are members of proposed ratio-type estimator \( T \) are more efficient than the other existing estimators \( T_j, (j = 0, 1, \ldots, 13) \) which are due to [1], [2], [12, 13, 14] and [3] respectively. From (2.16) and (3.14), we have

\[
\text{MSE} (T_k) < \text{MSE} (T_j) \text{ if } \tau_k^* (\tau_k^* - 2c) < \gamma_j (\gamma_j - 2c) \\
\text{i.e. if either, } \\
\tau_k^* < \gamma_j \text{ and } c < \left( \frac{\gamma_j + \gamma_j}{2} \right) \\
or, \\
\tau_k^* > \gamma_j \text{ and } c > \left( \frac{\gamma_j + \gamma_j}{2} \right) \\
or equivalently , \text{ min} [\gamma_j, (2c - \gamma_j)] \leq \tau_k^* \leq \text{ max} [\gamma_j, (2c - \gamma_j)], (j = 0, 1, \ldots, 13; k = 1, 2, \ldots, 6).
\]

5. Empirical study

The performance of the ratio-type estimators \( T_k, (k = 1, 2, \ldots, 6) \) which are members of the suggested ratio-type estimator \( T \) are evaluated against the usual unbiased estimator \( s_y^2 \) and the estimators \( t_j, (j = 1, 2, \ldots, 13) \) which are due to [1], [2], [12, 13, 14] and [3] respectively. for the population data set [Source: [4]] summarized in Table 1. We have computed the percent relative efficiencies (PREs) of the estimators \( t_j, (j = 1, 2, \ldots, 13) \) and the suggested ratio-type estimators \( T_k, (k = 1, 2, \ldots, 6) \) with respect to the usual unbiased estimator \( t_0 = s_y^2 \) in certain range of \( \alpha \in (0.0, 1.0) \) by using following formulae respectively as

\[
(5.1) \quad \text{PRE} (t_j, s_y^2) = \frac{\text{MSE} (s_y^2)}{\text{MSE} (t_j)} \times 100 = \frac{\lambda_{10}^*}{[\lambda_{10}^* + \gamma_j \lambda_{04}^* (\gamma_j - 2c)]} \times 100
\]

\[
(5.2) \quad \text{PRE} (T_k, s_y^2) = \frac{\text{MSE} (s_y^2)}{\text{MSE} (T_k)} \times 100 = \frac{\lambda_{10}^*}{[\lambda_{10}^* + \gamma_k^* \lambda_{04}^* (\gamma_k^* - 2c)]} \times 100
\]

and finding are summarized in Tables 2 and 3. It is observed from Tables 2 and 3 that all the ratio-type estimators \( T_k, (k = 1, 2, \ldots, 6) \) which are members of proposed ratio-type estimator \( T \) performed better than the usual unbiased estimator \( s_y^2 \), usual ratio estimator

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Table 1. The parameters of population data set

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>80</td>
</tr>
<tr>
<td>( n )</td>
<td>20</td>
</tr>
<tr>
<td>( \bar{Y} )</td>
<td>51.8264</td>
</tr>
<tr>
<td>( \bar{X} )</td>
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<tr>
<td>( \rho )</td>
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</tr>
<tr>
<td>( S_y )</td>
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</tr>
<tr>
<td>( C_y )</td>
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</tr>
<tr>
<td>( Q_1 )</td>
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</tr>
<tr>
<td>( S_x )</td>
<td>8.4563</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>10.300</td>
</tr>
</tbody>
</table>

Table 2. PREs of estimators \( t_j, (j = 0, 1, \ldots, 13) \) with respect to \( s_y^2 \)

<table>
<thead>
<tr>
<th>( t_0, s_y^2 )</th>
<th>( t_1, s_y^2 )</th>
<th>( t_2, s_y^2 )</th>
<th>( t_3, s_y^2 )</th>
<th>( t_4, s_y^2 )</th>
<th>( t_5, s_y^2 )</th>
<th>( t_6, s_y^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 100.00 )</td>
<td>183.23</td>
<td>179.24</td>
<td>181.98</td>
<td>164.49</td>
<td>226.87</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t_7, s_y^2 )</th>
<th>( t_8, s_y^2 )</th>
<th>( t_9, s_y^2 )</th>
<th>( t_{10}, s_y^2 )</th>
<th>( t_{11}, s_y^2 )</th>
<th>( t_{12}, s_y^2 )</th>
<th>( t_{13}, s_y^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 206.64 )</td>
<td>247.25</td>
<td>232.13</td>
<td>209.86</td>
<td>229.54</td>
<td>238.17</td>
<td>249.84</td>
</tr>
</tbody>
</table>
Table 3. PREs of estimators $T_k, (k = 1, 2, ..., 6)$ with respect to $s^2_y$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$(T_1, s^2_y)$</th>
<th>$(T_2, s^2_y)$</th>
<th>$(T_3, s^2_y)$</th>
<th>$(T_4, s^2_y)$</th>
<th>$(T_5, s^2_y)$</th>
<th>$(T_6, s^2_y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
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<td>183.23</td>
<td>183.23</td>
<td>183.23</td>
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<td>183.23</td>
</tr>
<tr>
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<td>195.20</td>
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</tr>
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</table>

$t_1$ due to [1] and the estimators $t_j, (j = 2, 3, 4, 5)$ due to [2] for all $\alpha \in (0.0, 1.0)$. However all the ratio-type estimators $T_k, (k = 1, 2, ..., 6)$ are more efficient than the estimators $t_j, (j = 6, 7, ..., 12)$ due to [12, 13, 14] and [3] for a specific value of $\alpha$. The estimators $T_2$ and $T_5$ which utilize the information on $(\beta_2(x), Q_2)$ and $(\rho, Q_d)$ respectively are the best in the sense of having largest percent relative efficiency among all the estimators discussed here for $\alpha = 1$.

6. Conclusion

In this paper we have suggested some ratio-type estimators of population variance $S^2_y$ of the study variable $y$ using known parameters of an auxiliary variable such as coefficient of variation, coefficient of kurtosis, correlation coefficient and quartiles etc. The bias and mean squared error formulae of the proposed ratio-type estimators are obtained and compared with that of the usual unbiased estimator, traditional ratio estimator and the estimators due to [2], [12, 13, 14] and [3]. We have also assessed the performance of the proposed estimators for known natural population data set and found that the performances of the proposed estimators are better than the other existing estimator for certain cases.

References


